Quantum noise frequency correlations of multiply scattered light

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Frequency correlations in multiply scattered light that are present in quantum fluctuations are investigated. The speckle correlations for quantum and classical noise are compared and are found to depend markedly differently on optical frequency, which was confirmed in a recent experiment. Furthermore, novel mesoscopic correlations are predicted that depend on the photon statistics of the incoming light. © 2006 Optical Society of America

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Light propagating through an optically thick disordered distribution of scattering particles induces an apparently random intensity pattern known as a volume speckle pattern. Despite the apparent randomness, such intensity speckle patterns can possess strong temporal and spatial correlations, giving rise to the so-called memory effect,[1] which manifests itself as a strong correlation between different spatial directions in a speckle pattern.[2,3] Similarly, strong frequency correlations exist in speckle patterns[4] that can be employed as a sensitive technique of measuring the diffusion constant of light propagation.[5] In addition to these relatively strong correlations, so-called mesoscopic intensity correlations also exist[6,7] that in general are weaker but can become dominating close to the transition to Anderson localization. Recently it was shown experimentally that novel speckle correlations exist when the fluctuations of light were recorded instead of merely the intensity.[8] Furthermore, strong spatial quantum correlations were predicted, depending on the quantum state of light incident on the medium.[9] In the present Letter a frequency correlation function is derived for noise, which was measured in Ref. 8, for an arbitrary quantum state of light. This work demonstrates that the quantum fluctuations of a volume speckle pattern give rise to strong correlations that differ from the correlations found for classical fluctuations of light or for intensities.

The propagation of photons through a multiple scattering random medium is studied using a full quantum model.[10] For convenience, the system is discretized into N different input modes and output modes. Let $\hat{n}_\omega^{ab}$ denote the photon number operator for light at the frequency ω propagating from input channel a to output channel b. The fluctuations are quantified by the photon number variance $(\Delta n_\omega^{ab})^2 = \langle(\hat{n}_\omega^{ab})^2\rangle - \langle\hat{n}_\omega^{ab}\rangle^2$, where the quantum mechanical expectation value will be evaluated for different quantum states of light. Further details about this model are given in Ref. 9. The quantum noise frequency correlation function for light propagating from input channel a to output channel b is defined by

$$C_{ab}^N(\Delta\omega) = \frac{\overline{(\Delta n_\omega^{ab})^2 \times (\Delta n_{\omega+\Delta\omega}^{ab})^2}}{\overline{(\Delta n_\omega^{ab})^2}^2} - 1. \tag{1}$$

This function measures the correlation between the fluctuations in a specific output direction for an optical frequency shift of $\Delta\omega$ and is the generalization of intensity correlation functions to the case of photon number fluctuations. The bars above the variances denote averaging over ensembles of disorder. We consider an arbitrary quantum state of light coupled through channel a characterized by an averaged number of photons $\langle\hat{n}_\omega^a\rangle$ and a Fano factor $F_a = (\Delta n_\omega^a)^2/\langle\hat{n}_\omega^a\rangle$. The photon number fluctuations of light transmitted to channel b are given by[9]

$$[(\Delta n_\omega^{ab})^2]_{QN} = \langle\hat{n}_\omega^a\rangle T_\omega^{ab} + \langle\hat{n}_\omega^a\rangle(F_a - 1)(T_\omega^{ab})^2, \tag{2}$$

where T_ω^{ab} is the intensity transmission coefficient from channel a to channel b. Here the subscript QN refers to quantum noise of the relevant quantum state. In the situation where classical noise (CN) is dominating, which, e.g., is the case when a laser beam is sent through an intensity modulator, the variance of the fluctuations is proportional to the squared average number of photons[8]:

$$[(\Delta n_\omega^{ab})^2]_{\mathrm{CN}} \propto \langle\hat{n}_\omega^a\rangle^2(T_\omega^{ab})^2. \tag{3}$$

Consequently, the correlation functions for quantum and classical noise are given by

$$C_{ab}^{\mathrm{QN}}(\Delta\omega) = \frac{\overline{T_\omega T_{\omega+\Delta\omega}} + (F_a - 1)\left[\overline{T_\omega^2 T_{\omega+\Delta\omega}} + \overline{T_\omega T_{\omega+\Delta\omega}^2}\right] + (F_a - 1)^2\overline{T_\omega^2 T_{\omega+\Delta\omega}^2}}{\overline{T_\omega}^2 + 2(F_a - 1)\overline{T_\omega} \times \overline{T_\omega^2} + (F_a - 1)^2\overline{T_\omega^2}^2} - 1, \tag{4}$$

$$C_{ab}^{\mathrm{CN}}(\Delta\omega) = \frac{\overline{T_{\omega}^2 T_{\omega+\Delta\omega}^2}}{\overline{T_{\omega}^2}^2} - 1, \tag{5}$$

where it has been assumed that $\langle \hat{n}_{\omega}\rangle = \langle \hat{n}_{\omega+\Delta\omega}\rangle$, corresponding to the situation where the input number of photons is kept constant while varying the optical frequency, which was the case in the experiment in Ref. 8. Note that indices ab on the transmission coefficients have been suppressed to simplify the notation.

In the lowest-order approximation, electric field transmission coefficient t_{ω} can be described by a circular Gaussian process, where $T_{\omega}=|t_{\omega}|^2$. The Gaussian approximation corresponds to omitting mesoscopic correlations that can subsequently be included perturbatively. For such Gaussian random variables, we have[11]

$$\overline{t_1^* \ldots t_k^* t_{k+1} \ldots t_{2k}} = \sum_{\pi} \overline{t_1^* t_p} \times \overline{t_2^* t_q} \times \ldots \times \overline{t_k^* t_r}, \tag{6}$$

where the summation is over all $k!$ permutations of the indices. Based on this theorem, we can evaluate the moments of T_{ω} that are relevant for Eqs. (4) and (5):

$$\overline{T_{\omega} T_{\omega+\Delta\omega}} = \overline{T_{\omega}}^2 + |\overline{t_{\omega}^* t_{\omega+\Delta\omega}}|^2, \tag{7}$$

$$\overline{T_{\omega}^2 T_{\omega+\Delta\omega}} = \overline{T_{\omega} T_{\omega+\Delta\omega}^2} = 2\overline{T_{\omega}}^3 + 4\overline{T_{\omega}} |\overline{t_{\omega}^* t_{\omega+\Delta\omega}}|^2, \tag{8}$$

$$\overline{T_{\omega}^2 T_{\omega+\Delta\omega}^2} = 4\overline{T_{\omega}}^4 + 16\overline{T_{\omega}}^2 |\overline{t_{\omega}^* t_{\omega+\Delta\omega}}|^2 + 4|\overline{t_{\omega}^* t_{\omega+\Delta\omega}}|^4, \tag{9}$$

$$\overline{T_{\omega}^n} = n!\,\overline{T_{\omega}}^n. \tag{10}$$

First, for the quantum noise, let us restrict ourselves to the special case of shot noise (SN), which corresponds to $F_a=1$. Hence, the correlation functions in Eqs. (4) and (5) are given by

$$C_{ab}^{\mathrm{SN}}(\Delta\omega) = \frac{|\overline{t_{\omega}^* t_{\omega+\Delta\omega}}|^2}{\overline{T_{\omega}}^2} \equiv f(\Delta\omega), \tag{11}$$

$$C_{ab}^{\mathrm{CN}}(\Delta\omega) = \frac{|\overline{t_{\omega}^* t_{\omega+\Delta\omega}}|^4 + 4\overline{T_{\omega}}^2 |\overline{t_{\omega}^* t_{\omega+\Delta\omega}}|^2}{\overline{T_{\omega}}^4} \equiv f^2(\Delta\omega) + 4f(\Delta\omega), \tag{12}$$

where

$$f(\Delta\omega) = \frac{\Delta\omega/\omega_D}{\cosh(\sqrt{\Delta\omega/\omega_D}) - \cos(\sqrt{\Delta\omega/\omega_D})} \tag{13}$$

is the function that describes the frequency decay of the intensity correlations[12] with $\omega_D = D/2L^2$.

The correlation functions for shot noise and technical noise are plotted in Fig. 1 as a function of frequency shift $\Delta\omega$. We observe that the noise correlations are much stronger in the case of classical noise relative to shot noise, thus the correlation at $\Delta\omega=0$ is a factor of 5 higher. This pronounced difference was verified experimentally in Ref. 8. We note that the correlation function for shot noise is identical to what one would get if one were recording the intensity, the latter being independent of the actual quantum state. Notably, the quantum fluctuations provide an independent measure of frequency speckle correlations, and, as will be seen in the following, the noise correlations depend strongly on the quantum state of light.

In the general case of an arbitrary quantum state characterized by the Fano factor F_a, the correlation function is given by Eq. (4). This expression contains terms of various orders in $\overline{T_{\omega}}$, where in general $\overline{T_{\omega}} \ll 1$; i.e., Eq. (4) can be expanded in orders of the intensity transmission coefficient. In the expansion of the products of transmission coefficients higher-order contributions must also be included that originate from deviations from Gaussian statistics of the transmission coefficients. The lowest-order contribution to these mesoscopic correlations can be calculated from an expansion in the inverse conductance $1/g = L/Nl$, which leads to[13,14]

$$\overline{T_{\omega} T_{\omega+\Delta\omega}} = \overline{T_{\omega}}^2 + |\overline{t_{\omega}^* t_{\omega+\Delta\omega}}|^2 + \frac{3L^2}{2l^2} g(\Delta\omega) \overline{T_{\omega}}^3, \tag{14}$$

where L is the thickness of the random medium, l is the transport mean free path, and we have defined the function

$$g(\Delta\omega) = \frac{\omega_D}{\Delta\omega} \times \frac{\sinh(\sqrt{\Delta\omega/\omega_D}) - \sin(\sqrt{\Delta\omega/\omega_D})}{\cosh(\sqrt{\Delta\omega/\omega_D}) - \cos(\sqrt{\Delta\omega/\omega_D})}. \tag{15}$$

Expanding Eq. (4) to first order leads to

$$C_{ab}^{QN}(\Delta\omega) \approx C_{ab}^{\mathrm{I}}(\Delta\omega) + C_{ab}^{\mathrm{II}}(\Delta\omega), \tag{16}$$

where

$$C_{ab}^{\mathrm{I}}(\Delta\omega) = f(\Delta\omega), \tag{17}$$

$$C_{ab}^{\mathrm{II}}(\Delta\omega) = \frac{3L^2}{2l^2} g(\Delta\omega)\overline{T_{\omega}} + 4(F_a - 1) f(\Delta\omega)\overline{T_{\omega}}. \tag{18}$$

The dominating term in the expansion (C_{ab}^{I}) is simply equal to the contribution obtained with shot noise.

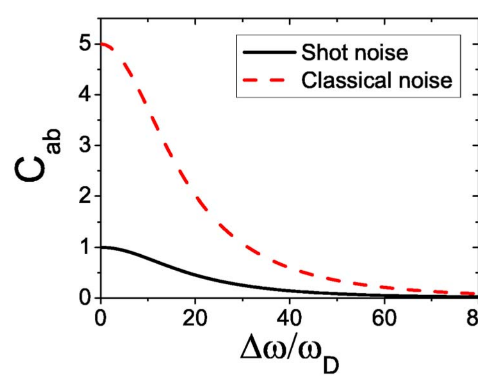

Fig. 1. (Color online) Noise correlation function for shot noise (solid curve) and classical noise (dashed curve) as a function of frequency shift.

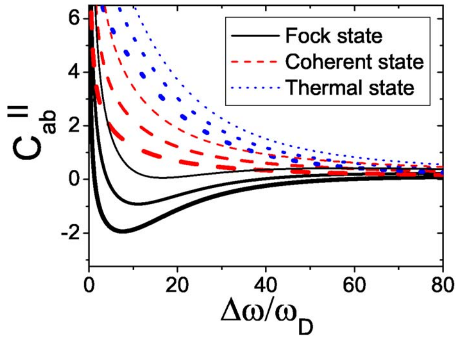

Fig. 2. (Color online) Second-order correlation function C_{ab}^{II} normalized to $\overline{T_\omega}$ as a function of frequency shift for three different quantum states of light: Fock state $F_a=0$, coherent state $F_a=1$, and thermal state $F_a=2$. The ratios of mean free path to the sample thickness are $l/L=1/3$ (thick curves), $l/L=1/4$ (medium curves), and $l/L=1/5$ (thin curves).

Deviations from shot noise behavior are observed when one is using quantum states of light different from the coherent state, i.e., $F_a \neq 1$, which give rise to the second term in Eq. (18). The quantum correction competes with classical mesoscopic correlations [first term in Eq. (18)], which is in contrast to the dominating quantum corrections found in the fluctuations of the total transmission and reflection.[9]

In Fig. 2, we plot the second-order noise correlation function $C_{ab}^{\mathrm{II}}(\Delta\omega)$ for three different quantum states of light, corresponding to $F_a=0$, $F_a=1$, and $F_a=2$. These Fano factors can be achieved with single-mode Fock states, coherent states, and thermal states, respectively. Figure 2 also indicates the correlation function for different ratios l/L, where $l/L \ll 1$ for optical media where multiple scattering dominates. The classical mesoscopic correlations identical to the ones obtained in intensity measurements are found for $F_a=1$. In the limit $\Delta\omega/\omega_D \to 0$, the classical correlation function diverges, which is a consequence of the plane-wave approximation and is suppressed when finite-width beams are included.[13] This sensitivity to the width of the beam plays an important role for only the correlations at $\Delta\omega/\omega_D \sim 1$. We observe from Fig. 2 that either positive or negative quantum noise correlations are obtained using either super-Poissonian ($F_a>1$) or sub-Poissonian photons ($F_a<1$), respectively. Consequently, the fluctuations are found to possess novel correlations depending on the quantum state of light. This is in sharp contrast to standard intensity correlations that are independent of the quantum state.

A novel noise correlation function for multiply scattered light has been introduced and evaluated for both classical noise and for arbitrary single-mode quantum states. Pronounced different correlations were found when comparing classical noise to quantum noise. Including higher-order correction terms in an expansion in the transmission coefficient, quantum corrections to the noise correlation function were predicted that have no analogy in classical intensity measurements.

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