

Quantum correlations induced by multiple scattering of quadrature squeezed light

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Abstract: Propagating quadrature squeezed light through a multiple scattering random medium is found to induce pronounced spatial quantum correlations that have no classical analogue. The correlations are revealed in the number of photons transported through the sample that can be measured from the intensity fluctuations of the total transmission or reflection. In contrast, no pronounced spatial quantum correlations appear in the quadrature amplitudes where excess noise above the shot noise level is found.

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OCIS codes: (290.4210) Multiple scattering; (270.6570) Squeezed states; (270.5290) Photon statistics

References and links

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1. Introduction

Multiple scattering of light is an interdisciplinary research field that connects traditionally remote areas such as optical communication [1] or acoustical [2] and biomedical [3] imaging to fundamental physics studies of Anderson localization of light [4, 5]. Most work on multiple scattering concerns situations where a classical description of both light and matter is sufficient [6]. A notable exception is the case of coherent backscattering from cold atomic clouds that can only be explained by implementing a quantum theory of matter [7]. Quantum optics studies of multiple scattering, where light is quantized, have only been initiated recently [8, 9, 10, 11, 12]. A surprisingly rich variety of new phenomena is encountered in the quantum optics theory, including the identification of spatial [9] and frequency [10] quantum correlations, and the information capacity transmitted through a multiple scattering medium [11]. Experimental work on transport of optical quantum noise was only initiated very recently [12].

The present paper concerns the transport of quadrature squeezed light through a multiple scattering random medium. Quadrature squeezed light can be generated in $\chi^{(2)}$ frequency conversion processes such as frequency doubling or optical parametric oscillation [13]. Previous work on scattering of squeezed light has been performed by Patra and Beenakker who studied the degradation of squeezing due to absorption and amplification in a random medium [8]. Here we investigate to what extent quantum correlations of squeezed light survive multiple scattering. The outcome is found to depend significantly on the parameters extracted. Thus, excess quantum fluctuations are encountered in the quadrature amplitudes, which can be traced back to the randomization of the optical phase from multiple scattering. In contrast, pronounced spatial quantum correlations can be found in the number of photons as was studied in [9] for the case of thermal states and Fock states. In the present manuscript spatial photon correlations are found to be induced by multiple scattering of quadrature squeezed light, and the nature of these correlations differs when comparing amplitude squeezed light to phase squeezed light.

2. Quantum optical model for multiple scattering

We describe the propagation of light through a multiple scattering random medium. The medium consists of a random distribution of particles that each have a high cross-section for light scattering. Such samples can be fabricated, e.g., by grinding or electrochemical etching of a high refractive index substrate [14]. The characteristic length scale that describes the multiple scattering strength is the transport mean free path ℓ . If the transport mean free path is much shorter than the thickness of the sample ($\ell \ll L$), light is multiply scattered in the medium. As a consequence, an incident wave will be diffused by the medium whereby the propagation direction is randomized. Hence the multiple scattering medium splits the incoming beam into a distribution of different output wavevectors that form a volume speckle pattern. In the extreme case where the transport mean free path becomes comparable to the optical wavelength ($\ell \sim \lambda$) light diffusion is significantly altered and eventually breaks down. In the present work we restrict to the case where multiple scattering is well-described by diffusion theory, which corresponds to the case of $\lambda \ll \ell \ll L$.

We wish to calculate the electric field of light that has been multiply scattered in the medium. Multiple scattering in the diffusive regime can be described as the result of many independent random events. We discretize the system and allow for N input k-vectors (labelled a') and N output k-vectors (labelled b'), cf. Fig. 1. The electric field in an arbitrary output 'channel'

$x = a$ or $x = b$ is found by summing over a number of partial waves, i.e. $\mathbf{E}_x(\mathbf{r}, t) = \sum_j \mathbf{A}_j e^{i\phi_j}$ [15]. From the central limit theorem it immediately follows that the multiple scattering statistics is Gaussian [16], which means that the electric field reflection and transmission coefficients $r_{a_ia_j} = \mathbf{E}_{a_j}^{out} / \mathbf{E}_{a_i}^{in}$ and $t_{a_ib_j} = \mathbf{E}_{b_j}^{out} / \mathbf{E}_{a_i}^{in}$ are complex Gaussian random variables. Hence $r_{a_ia_j} = \sqrt{\overline{R_{a_ia_j}}} e^{i\phi_{a_ia_j}}$ and $t_{a_ib_j} = \sqrt{\overline{T_{a_ib_j}}} e^{i\phi_{a_ib_j}}$ where the phase ϕ is uniformly distributed in the interval $[-\pi, \pi]$ while the intensity amplitudes R and T are described by Gaussian statistics. The ensemble averaged reflection and transmission coefficients are given by: $\overline{R_{a_ia_j}} = (1 - \ell/L) / N$ and $\overline{T_{a_ib_j}} = \ell / LN$ [6]. The uniform distribution of the phase implies $\overline{\phi_{a_ia_j}} = \overline{\phi_{a_ib_j}} = 0$.

The above discussion of the reflection and transmission holds only for a classical description. When describing quantum optical properties of light, such as the quantum fluctuations, the theory must be generalized. The difference between the quantum and classical theory is that in the former we explicitly have to include input from all open channels even the ones where no light is injected. Thus, the contributions of vacuum fluctuations in all open channels are crucial in quantum optics. The quantized electric field contains the annihilation and creation operators \hat{a} and \hat{a}^\dagger [13]. The annihilation operators for output channels a or b are related to the annihilation operator of all input channels through the expression [8, 9]

$$\hat{a}_b = \sum_{a'} t_{a'b} \hat{a}_{a'}^{in} + \sum_{b'} r_{b'b} \hat{a}_{b'}^{in}. \quad (1)$$

Here we use the notation that a denotes channels to the left and b channels to the right of the medium, c.f. Fig. 1. A similar expression can be given for the operator \hat{a}_a , which is obtained from Eq. (1) by substituting $r_{b'b} \rightarrow t_{b'a}$ and $t_{a'b} \rightarrow r_{a'a}$. The operators \hat{a}_a^{in} and $\hat{a}_{b'}^{in}$ describe the quantum state of all open channels, and even in the case where no light is coupled through a channel there will be an important contribution from vacuum fluctuations. The input annihilation and creation operators obey the commutation relations $[\hat{a}_{a'}^{in}, (\hat{a}_{a''}^{in})^\dagger] = \delta_{a', a''}$, $[\hat{a}_{b'}^{in}, (\hat{a}_{b''}^{in})^\dagger] = \delta_{b', b''}$, while all other commutation relations vanish. Note that Eq. (1) describes the very general situation of N input modes coupled to N output modes, while the specific characteristics of the multiple scattering random medium are contained in the reflection and transmission coefficients. Eq. (1) can be used to evaluate the propagation of any quantum state of light propagating through a lossless multiple scattering medium. In the present work we describe the particular case of squeezed light.

3. Transport of quadrature squeezed light through multiple scattering medium

The quadrature amplitude representation is relevant for describing quadrature squeezed light generated, e.g., by $\chi^{(2)}$ nonlinear processes. We introduce $\hat{x} = \hat{a} + \hat{a}^\dagger$ and $\hat{p} = -i(\hat{a} - \hat{a}^\dagger)$ that correspond to the real and imaginary components of the complex electric field. A quadrature minimum-uncertainty squeezed state can be described by the operators [13]

$$\hat{x}_s = x + e^{-s} \hat{x}_v, \quad (2)$$

$$\hat{p}_s = p + e^s \hat{p}_v, \quad (3)$$

where the operators \hat{x}_v and \hat{p}_v represent quadratures of the vacuum state, x and p are the classical amplitudes, and s is referred to as the squeezing parameter. Evaluating the quantum expectation values $\langle \hat{O} \rangle$ and the variance $(\Delta O)^2 = \langle \hat{O}^2 \rangle - \langle \hat{O} \rangle^2$, the input squeezed state is characterized by: $\langle \hat{x}_s \rangle = x$, $\langle \hat{p}_s \rangle = p$, $(\Delta x_s)^2 = e^{-2s}$, $(\Delta p_s)^2 = e^{2s}$, where it has been used that $\langle \hat{x}_v \rangle = \langle \hat{p}_v \rangle = 0$ and $(\Delta x_v)^2 = (\Delta p_v)^2 = 1$.

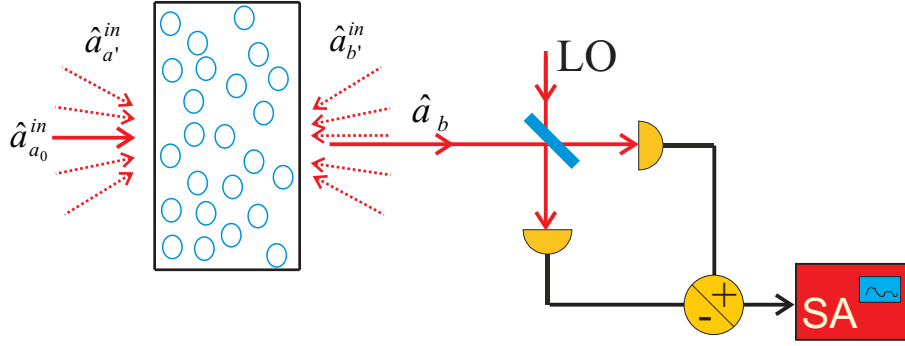


Fig. 1. Illustration of quadrature noise measurements of light transmitted through a multiple scattering medium. A squeezed state of light is injected through input channel a_0 while vacuum fluctuations contribute to all other open channels (indicated by dotted arrows). The quadratures amplitudes of the light transmitted to channel b can be measured by phase sensitive homodyning with a strong classical local oscillator (LO). By scanning the LO phase either the x or p quadrature can be obtained by subtracting the two photo currents, and their fluctuations can be recorded with a spectrum analyzer. The fluctuations in the photon number is obtained by blocking the LO and adding the two photocurrents.

Eq. (1) can be expressed in terms of the quadrature amplitudes relevant for describing quadrature squeezed light:

$$\hat{x}_b = \sum_{a'} \sqrt{T_{a'b}} [\cos \phi_{a'b} \hat{x}_{a'}^{in} - \sin \phi_{a'b} \hat{p}_{a'}^{in}] + \sum_{b'} \sqrt{R_{b'b}} [\cos \phi_{b'b} \hat{x}_{b'}^{in} - \sin \phi_{b'b} \hat{p}_{b'}^{in}], \quad (4)$$

$$\hat{p}_b = \sum_{a'} \sqrt{T_{a'b}} [\cos \phi_{a'b} \hat{p}_{a'}^{in} + \sin \phi_{a'b} \hat{x}_{a'}^{in}] + \sum_{b'} \sqrt{R_{b'b}} [\cos \phi_{b'b} \hat{p}_{b'}^{in} + \sin \phi_{b'b} \hat{x}_{b'}^{in}]. \quad (5)$$

Commutation relations imply $[\hat{x}_{b_i}, \hat{p}_{b_j}] = 2i\delta_{b_i, b_j}$, and by evaluating $[\hat{x}_{b_i}, \hat{p}_{b_j}]$ and $[\hat{x}_{b_i}, \hat{x}_{b_j}]$ explicitly, the following useful identities can be derived

$$\begin{aligned} & \sum_{a'} \sqrt{T_{a'b_i} T_{a'b_j}} [\cos \phi_{a'b_i} \cos \phi_{a'b_j} + \sin \phi_{a'b_i} \sin \phi_{a'b_j}] \\ & + \sum_{b'} \sqrt{R_{b'b_i} R_{b'b_j}} [\cos \phi_{b'b_i} \cos \phi_{b'b_j} + \sin \phi_{b'b_i} \sin \phi_{b'b_j}] = \delta_{b_i, b_j} \end{aligned} \quad (6)$$

$$\begin{aligned} & \sum_{a'} \sqrt{T_{a'b_0} T_{a'b_1}} [\sin \phi_{a'b_0} \cos \phi_{a'b_1} - \cos \phi_{a'b_0} \sin \phi_{a'b_1}] \\ & + \sum_{b'} \sqrt{R_{b'b_0} R_{b'b_1}} [\sin \phi_{b'b_0} \cos \phi_{b'b_1} - \cos \phi_{b'b_0} \sin \phi_{b'b_1}] = 0 \end{aligned} \quad (7)$$

In addition to quadrature amplitudes also the number of photons transported through the sample will be evaluated. The photon number operator is defined as $\hat{n} = \hat{a}^\dagger \hat{a}$ and using Eq. (1) leads to the relation

$$\hat{n}_b = \hat{a}_b^\dagger \hat{a}_b = \sum_{a', a''} t_{a'b}^* t_{a''b} \hat{a}_{a'}^\dagger \hat{a}_{a''} + \sum_{b', b''} r_{b'b}^* r_{b''b} \hat{a}_{b'}^\dagger \hat{a}_{b''} + \sum_{a', b'} [t_{a'b}^* r_{b'b} \hat{a}_{a'}^\dagger \hat{a}_{b'} + h.c.], \quad (8)$$

where “h.c.” means Hermitian conjugate. Similar expressions for the quadratures and photon number for channels a to the left of the medium could be equally stated. However, for brevity we will in the following sections detail only the calculations for transmission, while the corresponding reflections can be obtained by a equivalent calculation and the results will only be stated.

4. Quadrature amplitude correlations due to multiple scattering

Consider the situation where quadrature squeezed light is coupled through a single input channel denoted a_0 . We wish to evaluate the expectation value, quantum fluctuations, and spatial correlation for the light transmitted through the sample. The expectation value of the quadrature amplitude can be obtained experimentally from the average photocurrent in the homodyne detector of Fig. 1. The quantum fluctuations of a certain output channel can be measured from the fluctuations of the photocurrent while spatial correlations between two different output channels are revealed in the fluctuations of the total transmission [9].

4.1. Expectation values

In evaluating the expectation value of an observable in any output channel, only the contribution from the input coupling channel a_0 needs to be included. Thus, for the input vacuum channels $a' \neq a_0$ or b' , the expectation values vanish, i.e. $\langle \hat{x}_{a'}^{in} \rangle = \langle \hat{p}_{a'}^{in} \rangle = \langle \hat{x}_{b'}^{in} \rangle = \langle \hat{p}_{b'}^{in} \rangle = 0$, which by substitution into Eqs. (4-5) leads to

$$\langle \hat{x}_b \rangle = \sqrt{T_{a_0 b}} [\cos \phi_{a_0 b} \langle \hat{x}_{a_0}^{in} \rangle - \sin \phi_{a_0 b} \langle \hat{p}_{a_0}^{in} \rangle], \quad (9)$$

$$\langle \hat{p}_b \rangle = \sqrt{T_{a_0 b}} [\cos \phi_{a_0 b} \langle \hat{p}_{a_0}^{in} \rangle + \sin \phi_{a_0 b} \langle \hat{x}_{a_0}^{in} \rangle]. \quad (10)$$

These results are identical to the predictions of a purely classical theory for multiple scattering. It reflects the fact that quantum optical signatures are revealed in the *fluctuations* of light and *not in the expectation values*. Since the phase $\phi_{a_0 b}$ is a random variable, Eqs. (9-10) describe a spatial volume speckle pattern expressed in the quadrature amplitudes. Due to the random phase shift encountered in the multiple scattering process, the quadrature amplitudes vanish when averaging over all ensembles of disorder: $\langle \hat{x}_b \rangle = \langle \hat{p}_b \rangle = 0$, which is the well-known classical result [15]. In contrast, the ensemble averaged number of photons does not vanish as we will see in the following section.

4.2. Quantum fluctuations and spatial correlations

Quantum signatures of light are revealed, e.g., in the quantum fluctuations. Other inherent quantum signatures can be found from the temporal photon correlations that are measured in a Hanbury Brown-Twiss experiment [17], or through the spatial analogue where correlations between two different positions are considered [9]. In order to calculate the variance and the spatial correlation function we need to evaluate the product of two quadrature amplitudes in two channels b_i and b_j . The variance is obtained for $b_i = b_j$ and the spatial correlations for $b_i \neq b_j$. Eqs. (4) and (5) lead to

$$\begin{aligned} \langle \hat{x}_{b_i} \hat{x}_{b_j} \rangle &= \sum_{a'} \sqrt{T_{a' b_i} T_{a' b_j}} [\cos \phi_{a' b_i} \cos \phi_{a' b_j} \langle (\hat{x}_{a'}^{in})^2 \rangle - \cos \phi_{a' b_i} \sin \phi_{a' b_j} \langle \hat{x}_{a'}^{in} \hat{p}_{a'}^{in} \rangle \\ &\quad - \sin \phi_{a' b_i} \cos \phi_{a' b_j} \langle \hat{p}_{a'}^{in} \hat{x}_{a'}^{in} \rangle + \sin \phi_{a' b_i} \sin \phi_{a' b_j} \langle (\hat{p}_{a'}^{in})^2 \rangle] \\ &+ \sum_{b'} \sqrt{R_{b' b_i} R_{b' b_j}} [\cos \phi_{b' b_i} \cos \phi_{b' b_j} \langle (\hat{x}_{b'}^{in})^2 \rangle - \cos \phi_{b' b_i} \sin \phi_{b' b_j} \langle \hat{x}_{b'}^{in} \hat{p}_{b'}^{in} \rangle \\ &\quad - \sin \phi_{b' b_i} \cos \phi_{b' b_j} \langle \hat{p}_{b'}^{in} \hat{x}_{b'}^{in} \rangle + \sin \phi_{b' b_i} \sin \phi_{b' b_j} \langle (\hat{p}_{b'}^{in})^2 \rangle] \end{aligned}$$

$$\begin{aligned}
\langle \hat{p}_{b_i} \hat{p}_{b_j} \rangle &= \sum_{a'} \sqrt{T_{a'b_i} T_{a'b_j}} \left[\cos \phi_{a'b_i} \cos \phi_{a'b_j} \langle (\hat{p}_{a'}^{in})^2 \rangle + \cos \phi_{a'b_i} \sin \phi_{a'b_j} \langle \hat{p}_{a'}^{in} \hat{x}_{a'}^{in} \rangle \right. \\
&\quad \left. + \sin \phi_{a'b_i} \cos \phi_{a'b_j} \langle \hat{x}_{a'}^{in} \hat{p}_{a'}^{in} \rangle + \sin \phi_{a'b_i} \sin \phi_{a'b_j} \langle (\hat{x}_{a'}^{in})^2 \rangle \right] \\
&\quad + \sum_{b'} \sqrt{R_{b'b_i} R_{b'b_j}} \left[\cos \phi_{b'b_i} \cos \phi_{b'b_j} \langle (\hat{p}_{b'}^{in})^2 \rangle + \cos \phi_{b'b_i} \sin \phi_{b'b_j} \langle \hat{p}_{b'}^{in} \hat{x}_{b'}^{in} \rangle \right. \\
&\quad \left. + \sin \phi_{b'b_i} \cos \phi_{b'b_j} \langle \hat{x}_{b'}^{in} \hat{p}_{b'}^{in} \rangle + \sin \phi_{b'b_i} \sin \phi_{b'b_j} \langle (\hat{x}_{b'}^{in})^2 \rangle \right].
\end{aligned} \tag{11}$$

$$\tag{12}$$

As opposed to the calculation of the expectation values in the previous section, it is here crucial to include contributions from vacuum coupled through all open channels. Thus, vacuum fluctuations imply $\langle (\hat{x}_{a'}^{in})^2 \rangle = \langle (\hat{x}_{b'}^{in})^2 \rangle = \langle (\hat{p}_{a'}^{in})^2 \rangle = \langle (\hat{p}_{b'}^{in})^2 \rangle = 1$ for all $a' \neq a_0$ and b' . Furthermore, the product of quadrature amplitudes \hat{x} and \hat{p} can be evaluated identically for any of the input channel:

$$\langle \hat{x}_{b'}^{in} \hat{p}_{b'}^{in} \rangle = \langle \hat{x}_{a'}^{in} \hat{p}_{a'}^{in} \rangle = (-i) \langle (\hat{a}_{a'}^{in} + (\hat{a}_{a'}^{in})^\dagger) (\hat{a}_{a'}^{in} - (\hat{a}_{a'}^{in})^\dagger) \rangle = i \langle [\hat{a}_{a'}^{in}, (\hat{a}_{a'}^{in})^\dagger] \rangle = i, \tag{13}$$

and similarly $\langle \hat{p}_{a'}^{in} \hat{x}_{a'}^{in} \rangle = \langle \hat{p}_{b'}^{in} \hat{x}_{b'}^{in} \rangle = -i$.

Consider now a squeezed state coupled through channel a_0 , which is squeezed in the quadrature amplitude x , i.e. $\langle \hat{x}_{a_0}^{in} \rangle = x$, $\langle \hat{p}_{a_0}^{in} \rangle = p$, $\langle (\Delta \hat{x}_{a_0}^{in})^2 \rangle = \exp(-2s)$, $\langle (\Delta \hat{p}_{a_0}^{in})^2 \rangle = \exp(2s)$. These relations imply that $\langle (\hat{x}_{a_0}^{in})^2 \rangle = x^2 + e^{-2s}$ and $\langle (\hat{p}_{a_0}^{in})^2 \rangle = p^2 + e^{2s}$, which can be substituted into Eqs. (11) and (12) together with the above relations for the vacuum channels. We obtain

$$x_{b_i, b_j} = \delta_{b_i, b_j} + \sqrt{T_{a_0 b_i} T_{a_0 b_j}} \left[\cos \phi_{a_0 b_i} \cos \phi_{a_0 b_j} (e^{-2s} - 1) + \sin \phi_{a_0 b_i} \sin \phi_{a_0 b_j} (e^{2s} - 1) \right], \tag{14}$$

$$p_{b_i, b_j} = \delta_{b_i, b_j} + \sqrt{T_{a_0 b_i} T_{a_0 b_j}} \left[\sin \phi_{a_0 b_i} \sin \phi_{a_0 b_j} (e^{-2s} - 1) + \cos \phi_{a_0 b_i} \cos \phi_{a_0 b_j} (e^{2s} - 1) \right], \tag{15}$$

where $x_{b_i, b_j} \equiv \langle \hat{x}_{b_i} \hat{x}_{b_j} \rangle - \langle \hat{x}_{b_i} \rangle \langle \hat{x}_{b_j} \rangle$ and $p_{b_i, b_j} \equiv \langle \hat{p}_{b_i} \hat{p}_{b_j} \rangle - \langle \hat{p}_{b_i} \rangle \langle \hat{p}_{b_j} \rangle$. In deriving Eqs. (14) and (15) contributions from all open vacuum channels have been added up taking advantage of the identities of Eqs. (6) and (7). A special case is $b_i = b_j \equiv b$, where x_{b_i, b_j} and p_{b_i, b_j} correspond to the fluctuations in a single output channel

$$x_{b, b} \equiv \Delta x_b^2 = 1 + T_{a_0 b} (\cos^2 \phi_{a_0 b} e^{-2s} + \sin^2 \phi_{a_0 b} e^{2s} - 1), \tag{16}$$

$$p_{b, b} \equiv \Delta p_b^2 = 1 + T_{a_0 b} (\sin^2 \phi_{a_0 b} e^{-2s} + \cos^2 \phi_{a_0 b} e^{2s} - 1). \tag{17}$$

Eqs. (14) to (17) represent the cross-correlation and fluctuations for a single realization of a volume speckle pattern. As discussed above, the transmission coefficients $T_{a_0 b_i}$ and phases $\phi_{a_0 b_i}$ are random variables and therefore theory only can predict ensemble averaged quantities. Averaging over all ensembles of disorder leads to

$$\overline{\Delta x_b^2} = \overline{\Delta p_b^2} = 1 + \overline{T_{a_0 b}} (\cosh 2s - 1), \tag{18}$$

$$\overline{x_{b_i, b_j}} = \overline{p_{b_i, b_j}} = 0, \tag{19}$$

where $i \neq j$ in the latter equation. It is observed that the two-point correlation function vanishes for the quadrature amplitudes. Therefore, no spatial quantum correlations will be observed

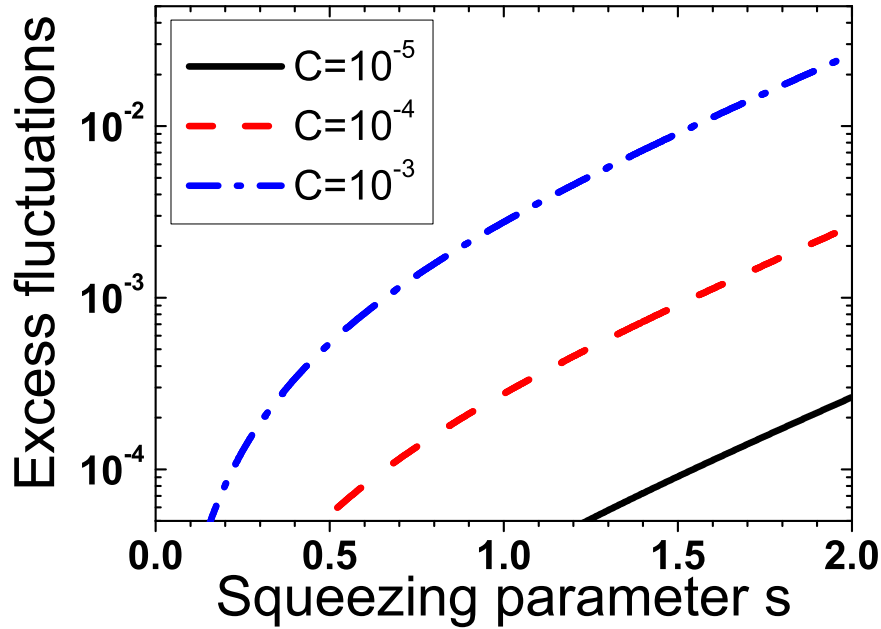


Fig. 2. Excess fluctuations above the shot noise level defined as $\frac{\overline{\Delta x_b^2}}{(\Delta x_b^2)_{SN}} - 1$ as a function of the squeezing parameter s . The excess fluctuations increase when $C \equiv \ell/NL$ and the squeezing parameter s are increased.

when measuring quadrature amplitudes, as opposed to the pronounced photon number spatial correlations first studied in [9] and also considered in the following section in the case of quadrature squeezed light. While no spatial correlations are found, the squeezed light leads to enhanced noise in each individual output channel. Curiously equal excess noise is obtained in both the squeezed and anti-squeezed quadratures, which is a consequence of the random phase response of the multiple scattering medium. The shot-noise level (SN) is defined as the variance of a coherent state ($s = 0$), i.e. $(\Delta x_b^2)_{SN} = 1$. A squeezed input is seen to lead to excess fluctuations above the shot noise level in both quadratures, i.e.

$$\frac{\overline{\Delta x_b^2}}{(\Delta x_b^2)_{SN}} = 1 + \frac{\ell}{N \times L} (\cosh 2s - 1). \quad (20)$$

These excess fluctuations are in general relatively weak. Thus, the prefactor determining their strength is $C \equiv \ell/NL = \ell^2/L^2 \times 1/g$ where $g = N\ell/L$ is the mesoscopic conductance that is much less than unity except for close to the Anderson localization transition, which is not considered here. Figure 2 shows the excess fluctuations above the shot-noise level as a function of the squeezing parameter s for three typical values of C [18].

5. Photon correlations induced by multiple scattering of quadrature squeezed light

In this section we will focus on the fluctuations and correlations in the number of photons that are multiply scattered into a certain channel for the case of quadrature squeezed light. Photon number fluctuations can be recorded in the intensity noise of light transmitted or reflected through the sample. As will be shown below, the spatial correlation function can be obtained from the fluctuations of the total reflection or transmission from the multiple scattering medium.

The photon number operator can be expressed in terms of quadrature amplitudes as $\hat{n} = (\hat{x}^2 + \hat{p}^2) / 4 - 1/2$. Thus, the photon number fluctuations can in principle be derived from the expressions for \hat{x}_b and \hat{p}_b used in the previous section. However, this appears not to be the most efficient strategy since it would require evaluation of quartic operator products. Instead it is more economic to follow the procedure outlined in [9] where the photon number operators are equated directly from Eq. (1). This leads to

$$\hat{n}_b = \hat{a}_b^\dagger \hat{a}_b = \sum_{a', a''} t_{a'b}^* t_{a''b} \hat{a}_{a'}^\dagger \hat{a}_{a''} + \sum_{b', b''} r_{b'b}^* r_{b''b} \hat{a}_{b'}^\dagger \hat{a}_{b''} + \sum_{a', b'} \left[t_{a'b}^* r_{b'b} \hat{a}_{a'}^\dagger \hat{a}_{b'} + h.c. \right], \quad (21)$$

where ‘‘h.c.’’ means Hermitian conjugate. Similarly the expression for $\hat{n}_{b_i} \hat{n}_{b_j}$ can be derived and contains in total 16 different terms. Restricting to the case of light injected through the single channel (a_0), only a few terms do not vanish when calculating the expectation value, which implies that [9]

$$\langle \hat{n}_b \rangle = T_{a_0 b} \langle \hat{n}_{a_0}^{in} \rangle, \quad (22)$$

$$n_{b_i, b_j} = \left[T_{a_0 b_i} \delta_{b_i, b_j} + T_{a_0 b_i} T_{a_0 b_j} (F_{a_0}^{in} - 1) \right] \langle \hat{n}_{a_0}^{in} \rangle, \quad (23)$$

where $n_{b_i, b_j} \equiv \langle \hat{n}_{b_i} \hat{n}_{b_j} \rangle - \langle \hat{n}_{b_i} \rangle \langle \hat{n}_{b_j} \rangle$ and the Fano factor $F_{a_0}^{in} = (\Delta n_{a_0}^{in})^2 / \langle \hat{n}_{a_0}^{in} \rangle$ measures the photon number fluctuations of the input light. Taking $i = j$ in Eq. (23) gives the variance of the fluctuations while $i \neq j$ gives the spatial correlation function, i.e.

$$\Delta n_b^2 = [T_{a_0 b} + T_{a_0 b}^2 (F_{a_0}^{in} - 1)] \langle \hat{n}_{a_0}^{in} \rangle, \quad (24)$$

$$n_{b_i, b_j} = T_{a_0 b_i} T_{a_0 b_j} (F_{a_0}^{in} - 1) \langle \hat{n}_{a_0}^{in} \rangle, \quad (25)$$

where the latter equation only holds for $b_i \neq b_j$.

The spatial correlations contribute significantly to the fluctuations of the total reflection and transmission. In the following we will again for brevity concentrate on the case of transmission while reflection can be treated equally. The total number of transmitted photons are found by summing over all transmission channels b , i.e. $\hat{n}_T = \sum_b \hat{n}_b$ and the variance is given by

$$\Delta n_T^2 = \sum_b \Delta n_b^2 + \sum_{b_i} \sum_{b_j \neq b_i} n_{b_i, b_j}. \quad (26)$$

Clearly the variance of the total number of transmitted photons is not just equal to the sum of the variances of each of the individual conduction channels. Hence a cross-term that describes spatial two-point correlations between different channels contributes to the fluctuations, and the quantum noise provides a valuable tool of measuring such non-trivial correlations.

The fluctuations of the total transmission of a squeezed beam is quantified relative to the fluctuations of a coherent state, i.e.

$$S_T = \frac{\left(\overline{\Delta n_T^2} \right)_{F_{a_0}}} {\left(\overline{\Delta n_T^2} \right)_{F_{a_0}=1}} = \frac{\sum_{b_i} \left[\overline{T_{a_0 b_i}} + \overline{T_{a_0 b_i}^2} (F_{a_0}^{in} - 1) \right] + \sum_{b_i} \sum_{b_j \neq b_i} \overline{T_{a_0 b_i} T_{a_0 b_j}} (F_{a_0}^{in} - 1)} {\sum_{b_i} \overline{T_{a_0 b_i}}}, \quad (27)$$

where the average over ensembles of disorder has been carried out.

As opposed to the case of quadrature amplitudes, discussed in the previous section, here products of intensity transmission coefficients are seen to appear. The ensemble average of these products are within the Gaussian approximation given by [6, 19] $\overline{T_{a_0 b_i}^2} = 2\ell^2/L^2N^2$ and $\overline{T_{a_0 b_i}T_{a_0 b_j}} = \ell^2/L^2N^2$, where in the latter case $i \neq j$. Neglecting a term of order $1/N$, which is of the same order of magnitude as non-Gaussian correction terms, leads to

$$S_T = 1 + \frac{\ell}{L} (F_{a_0}^{in} - 1). \quad (28)$$

For a discussion of the $1/N$ corrections that only play a dominant role close to the transition to Anderson localization, see [9]. Through an equivalent calculation also the fluctuations of the total reflections can be derived, which leads to

$$S_R = 1 + \left(1 - \frac{\ell}{L}\right) (F_{a_0}^{in} - 1). \quad (29)$$

Consequently, the total transmission and reflection of squeezed light is completely determined by the mean free path ℓ , the sample thickness L and the Fano factor of the incoming beam. For a quadrature squeezed state $\hat{x}_{a_0}^{in} = x + \exp(-s)\hat{x}_v$, $\hat{p}_{a_0}^{in} = p + \exp(s)\hat{p}_v$ it is straightforward to show that the Fano factor is given by

$$F_{a_0}^{in} = \frac{p^2 e^{2s} + x^2 e^{-2s} + \cosh 4s - 1}{x^2 + p^2 + 2(\cosh 2s - 1)}. \quad (30)$$

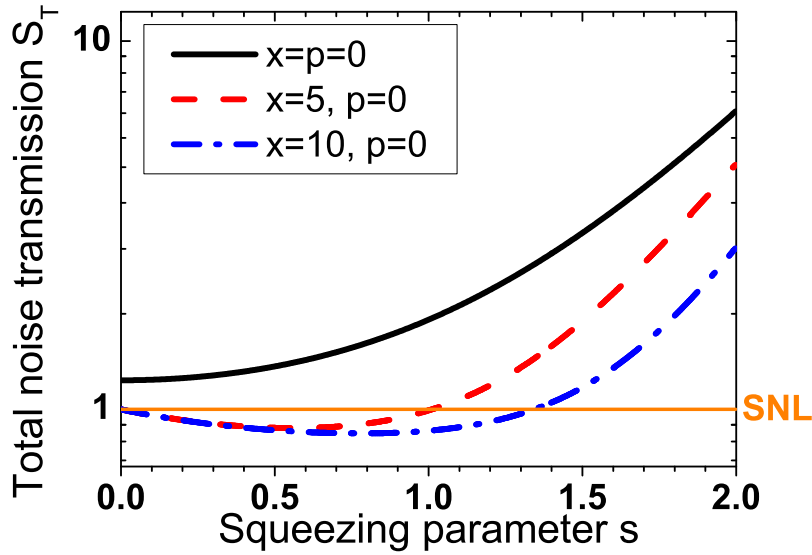


Fig. 3. Total transmission of noise as a function of the squeezing parameter s for quadrature amplitude-squeezed light and $\ell/L = 0.2$.

Now consider the photon number fluctuations of the total transmission for various squeezed beams. Figure 3 investigates the case of amplitude-squeezed light where $p = 0$, i.e. the classical

amplitude of the anti-squeezed quadrature vanishes. Figure 4 treats the phase-squeezed case where $x = 0$ such that the classical amplitude of the squeezed quadrature vanishes. In the figures the shot noise level (SNL) is indicated, which is the noise level for a coherent state, i.e. for $s = 0$. In the amplitude-squeezed case, the fluctuations of the total transmission are reduced below SNL for moderate values of the squeezing parameter s , c.f. Fig. 3. This implies that negative spatial correlations are induced between photons transmitted to different output channels, as first pointed out in [9] for the case of Fock states. Increasing the squeezing parameter for a fixed classical amplitude leads to increased fluctuations above SNL, which corresponds to positive spatial correlations resembling what was found for thermal light [9]. In the case of quadrature phase squeezing, shown in Fig. 4, excess fluctuations above SNL is found for all finite values of the squeezing parameter, i.e. positive spatial correlations are induced. A special case is $x = p = 0$ corresponding to a squeezed vacuum state with no classical amplitude. In this case excess noise above SNL is obtained for all s , which is a direct consequence of the fact that the photon statistics of a squeezed vacuum state always is super-Poissonian, i.e. $F > 1$.

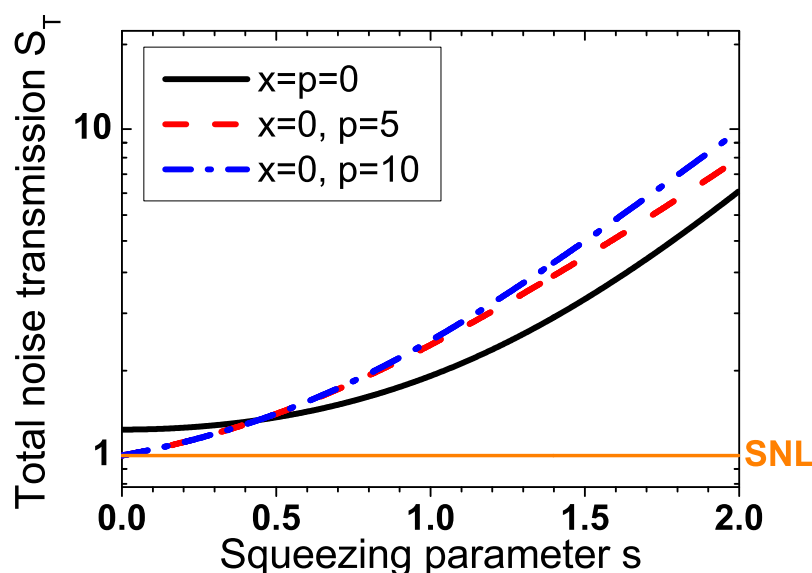


Fig. 4. Total transmission of noise as a function of the squeezing parameter s for quadrature phase-squeezed light and $\ell/L = 0.2$.

6. Conclusions

The quantum optics theory of multiple scattering was used to describe squeezed light. Focusing on quadrature squeezed states, a pronounced difference was encountered when analyzing the fluctuations of quadrature amplitudes as opposed to the photon number fluctuations. In the former case excess fluctuations above the shot noise level arose in each output channel while no spatial quadrature correlations were found. In contrast, pronounced spatial correlations are induced in the number of photons that can be either positive or negative depending on the squeezing parameter s and the classical amplitude of the squeezed light.

Acknowledgments

I would like to thank Ad Lagendijk for stimulating discussions and gratefully acknowledge the Danish Research Agency (division FNU) for financial support.