Abstract

I focus on the scalar one-point functions in $SO(6)$ sector of D5-D3 probe-brane set-up. Start with a general introduction of integrability, I explore both coordinate Bethe ansatz and algebraic Bethe ansatz, with possible generalization. I then shortly review how to use the Bethe ansatz in $\mathcal{N}=4$ super Yang-Mills theory, and then apply such procedure to the D5-D3 system. The dual field theory of such system corresponds to a defected version of $\mathcal{N}=4$ super Yang-Mills theory, where the one-point functions of certain scalars are non-zero. The calculation of one-point functions is mapped to find the overlap between matrix product states and Bethe states. The matrix product states are found to be solutions of the twisted Boundary Yang-Baxter equation, and equivalently the representations of extended twisted Yangian. By dressing procedure or coproduct property, we can connect the scalar matrix product state and higher dimension matrix product states. We have used the branching rules to find the connection, but there are some detailed parameters needed to be fixed. Such method can not only be used for calculations of one-point functions in probe-branes system, but also shed some light on non-equilibrium system.