Entangled Particles

Alexander Bschorr • B. Sc. Student • Copenhagen University • Spring/Summer 2010

Introduction

Throughout history human interpretation and understanding of nature and its phenomena has changed many times.

Before modern science was born, it was common to explain observed phenomena by means of mythological beings (as it is still practiced by primitive peoples today). A vast number of gods, goddesses, ghosts, demons and other supernatural beings were believed to be responsible for observations in nature.

This view changed more and more with the appearance of proper science. Nature basically was demystified. Northern Lights, for example, were discovered to be solar winds hitting the atmosphere, and also earthquakes, tsunamis or volcano eruptions were found to have physical causes that could be well understood, and which even made the prediction of such devastating events partly possible.

The demystification of natural phenomena was a long process, and it also did not hinder people from keeping, when also modifying, their beliefs in supernatural beings. Newton himself, after having found out which laws govern the movement of bodies in the universe, believed that God had set the universe in the state we find it to be. Still today people argue for that God created the universe at time of the Big Bang.

But unlike the debates about the origin of the universe, there was a broader and broader consensus among scientists that nature always behaves in a deterministic way. Especially with the explosion of knowledge in natural science during the second part of the 19th century, most scientists became convinced about nature´s deterministic behavior, as all new discoveries validated this view.

Therefore, it is not surprising that the discovery of quantum mechanics in the beginning of the 20th century, seeing nature as indeterminate on the quantum level, led to a vast number of discussions among scientists about whether quantum mechanics is complete, or not. The most relevant of these discussions was about the phenomena of *entanglement*. It started in the 1930s and lasted in its essence until the 1990s.

In the following, we will examine entanglement and the historical development of the debate over the causes of entanglement. In most common introductory textbooks, these topics are only touched shortly, and results therefore rather named than explicitly deduced, whereas scientific articles mainly deal with a specific property of entanglement, and require a deep basic knowledge in order to be fully understood. This leaves undergraduate physics students with a superficial knowledge about entanglement. In this paper, we will analyze crucial points in detail, and derive important equations explicitly in a way understandable to undergraduate physics students. Furthermore, will we discuss at a recent experiment about entanglement in which the minimum speed for a communication between entangled particles was found to be 10,000 faster than light. Finally, we briefly study the best known alternative interpretation of quantum mechanics that tries to keep a deterministic world view.

Main Part

The statistical interpretation of the wave function and indeterminacy in the Copenhagen interpretation of quantum mechanics

In the Copenhagen interpretation of quantum mechanics, it is assumed that the physical state of a quantum system is completely described by the wave function. The wave function enables one to predict the statistical probability to attain a certain value in a measurement of the system. Hereby, it is crucial that we see it as a property of nature that all we can say about nature is statistics, and not our inability to understand deeper, the behavior of particles guiding principles. We therefore can not explain why or how measurement outcomes ("events") appear as we find them to be.

The assumption that all we can say about nature is statistics has always been criticized by physicists (a. o. by Einstein) who believed that even at the quantum level, there must exist precisely definable dynamical variables determining (as in Classical physics) the actual behavior of each individual system, and not merely its probable behavior. Since these variables are not included in quantum theory, Einstein has always regarded the present form of quantum mechanics as incomplete.

The Einstein-Podolsky-Rosen paradox

In 1935 Albert Einstein, Boris Podolsky and Nathan Rosen published a four-sided article in the science magazine "Physical Review" in which they argued on the base of a *Gedanken experiment* that the description of quantum mechanics is not complete.[1] The content of the article became known, and still is referred to, as the *Einstein-Podolsky-Rosen paradox* (EPR), although it is does not describe a real paradox.

We will here examine a simplified version introduced by David Bohm[2].

The phenomena

We consider the decay of a particle at rest with Spin 0 into two particles with Spin ½, for example, a η-meson decay into two muons, or a π-meson decay into an electron and a positron. According to the conservation of momentum, the two particles move in opposite direction[I], and due to the conservation of angular momentum, the only describable state for them is the *singlet configuration*[II]:

$$S=0 \qquad \mathrm{I}\, Spin\ singlet >= \frac{1}{\sqrt{2}} \left(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\ \right) \qquad (0)$$

If we measure the spin at A or B (Fig. 1) in an arbitrary direction, but with parallel oriented detectors at A and B, we will find it to be with 50% probability in "up" or "down" direction.[III] We consider now that we measure the Spin in z-direction and find it to be "up" in A. Now we

---

[I] They move and do not rest as some of the mass of the mother particle is transformed into energy obliging $E=mc^2$.

[II] Remember: in a non-entangled state, we could write the wave-function (and also its spin part) of a two particle system as a product of the two single wave functions, $\Psi_{1,2} = \psi_1\psi_2$,and could clearly differ in what state particle 1 and 2 are. As a fundamental feature of entangled states, that is not possible here. We can not tell in what separate independent states particle 1 and 2 are. They are entangled.

[III] "Up"/"down" means that the spin functions have *eigenvalue* $\pm\frac{h}{2}$.

immediately know that it must be “down” in B, as the sum must be zero, no matter how far the particles are apart. The spin of the two particles seems to be correlated.
If we, however, orient the detectors differently, let us say, we measure the spin in x-direction at A, and in z-direction at B, than no correlation can be found.[3]

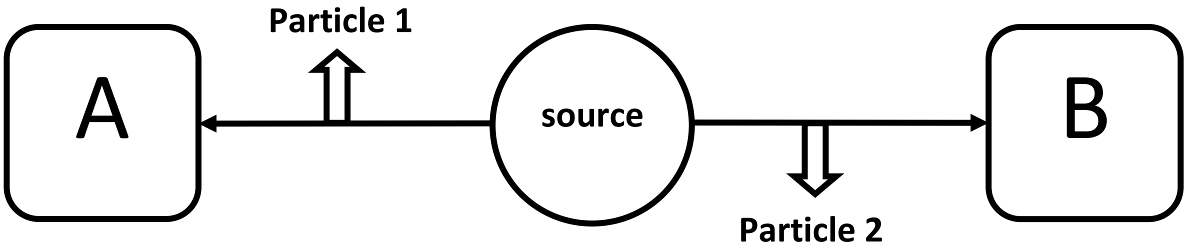

Figure 1: Spin correlation in a singlet state. A source produces pairs of entangled particles that are sent in opposite direction to detection stations at A and B. A measurement of the same spin component at A and B leads to correlated values. The sum of the spin at A and B is always zero. Therefore, we can predict the value of the second particle by measuring the first. When measuring a different spin component at A than at B, no correlation is found.

EPR´s believed paradox

For Einstein, Podolsky and Rosen, as for all supporters of the deterministic world view, the principles of *realism* and *locality* were applied to all of nature.
In the context of the experiment, *realism* states that the result of a measurement on a particle describes properties that the particle had before and independent of the measurement.
Locality means that no influence can travel faster than light and here also that results of measurements performed on one particle must be independent of whatever is done at the same time to the other particle which is located an arbitrary distance away.

The idea that the outcome of a spin measurement at the second particle could depend on the measurement of the first was thus completely unthinkable for Einstein, Podolsky and Rosen. In their article they claimed: “No reasonable definition of reality could be expected to permit this”[4]. Einstein earlier stated: “But to one supposition we should, in my opinion, hold absolutely fast: The real factual situation of the system S2 is independent of what is done with the system S1, which is spatially separated from the former. “[5].
That, according to quantum mechanics, the second particle´s spin is instantaneously known when the one of the first particle is measured, Einstein therefore called “spooky action at a distance”.

In Einstein, Podolsky and Rosen´s view, the information about the spin must have been in the particles all time from their decay to the measurement. But as named before, according to the Copenhagen interpretation, all we could say about the spin of a particle before the measurement is statistics. The fact that the wave function does not certainly determine the outcome of an individual measurement was believed to be the proof that quantum mechanics is incomplete and lacks a “hidden” variable. This hidden variable should give a particle a clearly predictable behavior for each point of time.

In the years after the EPR-paper, several hidden variable theories were proposed that should make the same predictions as quantum mechanics, without giving up the classical worldview. The passionate debate about whether quantum mechanics (or physics in general) describes nature “as it really is”, or “what we can say about nature” went on, but stayed rather on a philosophical level.
That changed in 1964, when the Northern Irish physicist John S. Bell showed that any local hidden variable theory produces different predictions than quantum mechanics. In his paper, Bell derived an inequality-relation connected to the spin measurement of entangled particles that must be valid for every local hidden variable theory and which is in direct contradiction to the predictions of quantum mechanics.

Now the question was no longer if quantum mechanics is complete or not, but if it is right at all, and it could be tested.

Bell´s inequality

Bell modified the earlier presented experiment of spin measurement of entangled particles to have freely rotating detectors (pointing in direction of the unit vectors $\boldsymbol{a}$ at A, and $\boldsymbol{b}$ at B).
As stated earlier, the supporter of the realist and local worldview have proposed a hidden variable that governs the particle`s motion. Bell describes this (supposedly) more complete formulation of quantum mechanics by the continuous parameter λ. Furthermore, he relates the spin measurement results $\boldsymbol{\sigma^1} \cdot \boldsymbol{a}$ and $\boldsymbol{\sigma^2} \cdot \boldsymbol{b}$ ($\boldsymbol{\sigma^1}$ and $\boldsymbol{\sigma^2}$ are the pauli-matrices of the spin operators of particle 1 and 2) to the functions $A(\boldsymbol{a},\lambda)$ and $B(\boldsymbol{b},\lambda)$ which can only take the values ± 1. When the detectors are parallel, meaning $\boldsymbol{a} = \boldsymbol{b}$ than:

$$A(\boldsymbol{a},\lambda) = -\mathrm{B}(\boldsymbol{a},\lambda). \tag{1}$$

Bell further assumes that the result in B does not depend on the measurement in A. For realizing that he suggests fixing the detectors` orientations first a tiny moment before the particles reach the detectors, so that no subluminal communication could take place.
Using the hidden variable λ and its probability distribution $\rho(\lambda)$ [IV], the expectation value of the product of the two components is:

$$E(\boldsymbol{a},\boldsymbol{b}) = \int \rho(\lambda) A(\boldsymbol{a},\lambda) B(\boldsymbol{b},\lambda)\, d\lambda \tag{2}$$ [V]

This should equal the quantum mechanical expectation value as hidden variable theories are supposed to make the same predictions. But as a matter of fact, the quantum mechanical expectation value is:

$$E(\boldsymbol{a},\boldsymbol{b})^{quantum\ mechanics\ (qm)} = < (\boldsymbol{\sigma^1} \cdot \boldsymbol{a})(\boldsymbol{\sigma^2} \cdot \boldsymbol{b}) > = -\boldsymbol{a} \cdot \boldsymbol{b} \tag{3}$$

[IV] As λ is continuous and normalized distributed meaning: $\int \lambda \, d\lambda = 1$.

[V] The possibility of having two sets of λs, one governing A and one B is included.

Proof:

For simplicity, we choose that **a** lies on the z-axis, and **b** in the xz-plane (Fig. 2).

This means:

$$\boldsymbol{\sigma}^{1} \cdot \boldsymbol{a} = \boldsymbol{\sigma}_{z}^{1} \text{ and } \boldsymbol{\sigma}^{2} \cdot \boldsymbol{b} = cos\theta_{ab}\boldsymbol{\sigma}_{z}^{2} + \sin\theta_{ab}\boldsymbol{\sigma}_{x}^{2}$$

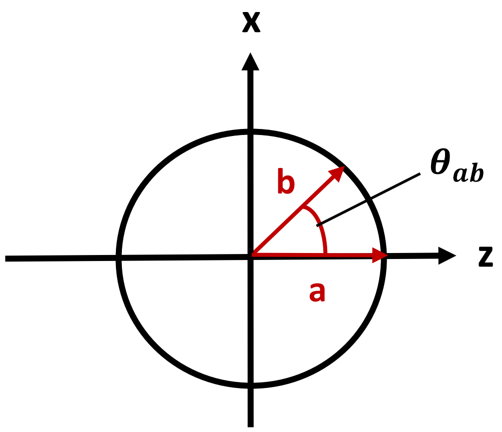

Figure 2: A possible detector orientation. The first detector is directed in z-direction, the second´s orientation lies in the xz-plane

We remember that the general state of a spin ½ particle can be written as:

$$\chi = a\chi_{+} + b\chi_{-} = a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix}. \quad (4)$$

$\chi_{+}/\,\chi_{-}$ stand for “spin up” and “spin down”.

Therefore does $\boldsymbol{\sigma}_{z} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ not change the “spin up”-vector, but changes the sign of “spin down”-vector, whereas $\boldsymbol{\sigma}_{x} = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ changes “up” to “down” and vice versa.

We now go back to the expectation value that we want to calculate:

$$< (\boldsymbol{\sigma}^{1} \cdot \boldsymbol{a})(\boldsymbol{\sigma}^{2} \cdot \boldsymbol{b}) > = < \text{Spin singlet I } (\boldsymbol{\sigma}^{1} \cdot \boldsymbol{a})(\boldsymbol{\sigma}^{2} \cdot \boldsymbol{b}) \text{ I Spin singlet} >$$

We start with:

$$(\boldsymbol{\sigma}^{1} \cdot \boldsymbol{a})(\boldsymbol{\sigma}^{2} \cdot \boldsymbol{b}) \text{ I Spin singlet} > = \boldsymbol{\sigma}_{z}^{1}(cos\theta_{ab}\boldsymbol{\sigma}_{z}^{2} + \sin\theta_{ab}\boldsymbol{\sigma}_{x}^{2}) \text{ I}\frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\ ) >$$

$$= \frac{1}{\sqrt{2}}[(\boldsymbol{\sigma}_{z}^{1}\uparrow_1)(cos\theta_{ab}\boldsymbol{\sigma}_{z}^{2}\downarrow_2 + \sin\theta_{ab}\boldsymbol{\sigma}_{x}^{2}\downarrow_2) - (\boldsymbol{\sigma}_{z}^{1}\downarrow_1)(cos\theta_{ab}\boldsymbol{\sigma}_{z}^{2}\uparrow_2 + \sin\theta_{ab}\boldsymbol{\sigma}_{x}^{2}\uparrow_2)]$$

$$= \frac{1}{\sqrt{2}}\{(\uparrow_1)[cos\theta_{ab}(-\downarrow_2) + \sin\theta_{ab}(\uparrow_2)] - (-\downarrow_1)[cos\theta_{ab}(\uparrow_2) + \sin\theta_{ab}(\downarrow_2)]\}$$

$$= cos\theta_{ab}\frac{1}{\sqrt{2}}(-\uparrow_1\downarrow_2 + \downarrow_1\uparrow_2) + sin\theta_{ab}\frac{1}{\sqrt{2}}(\uparrow_1\uparrow_2 + \downarrow_1\downarrow_2\ ) \quad (5)$$

Now:

$$< \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\ ) \text{ I } (\boldsymbol{\sigma}^{1} \cdot \boldsymbol{a})(\boldsymbol{\sigma}^{2} \cdot \boldsymbol{b}) \text{ I}\frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\ ) >=$$

$$=< \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\ ) \text{ I I } - cos\theta_{ab}\frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2) + sin\theta_{ab}\frac{1}{\sqrt{2}}(\uparrow_1\uparrow_2 + \downarrow_1\downarrow_2\ ) >$$

$$= -cos\theta_{ab} < \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\ ) \text{ I I}\frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\ ) > = -cos\theta_{ab} \quad (6)$$

As **a** and **b** are unit vectors, we can see that:

$$E(\boldsymbol{a},\boldsymbol{b})^{quantum\ mechanics\ (qm)} = -cos\theta_{ab} = -\boldsymbol{a}\cdot\boldsymbol{b}\ \ QED$$

This result is incompatible with the one for hidden variable theories. For the case of parallel detectors (1), we can rewrite equation (2) in the following way:

$$E(\boldsymbol{a},\boldsymbol{b}) = -\int \rho(\lambda)A(\boldsymbol{a},\lambda)A(\boldsymbol{b},\lambda)\,d\lambda \tag{7}$$

Now, we take another unit vector $\boldsymbol{c}$ and calculate:

$$E(\boldsymbol{a},\boldsymbol{b}) - E(\boldsymbol{a},\boldsymbol{c}) = -\int \rho(\lambda)A(\boldsymbol{a},\lambda)A(\boldsymbol{b},\lambda)\,d\lambda - (-)\int \rho(\lambda)A(\boldsymbol{a},\lambda)A(\boldsymbol{c},\lambda)\,d\lambda$$

$$= -\int \rho(\lambda)[A(\boldsymbol{a},\lambda)A(\boldsymbol{b},\lambda) - A(\boldsymbol{a},\lambda)A(\boldsymbol{c},\lambda)]d\lambda$$

$$= -\int \rho(\lambda)[1 - A(\boldsymbol{b},\lambda)A(\boldsymbol{c},\lambda)]\,A(\boldsymbol{a},\lambda)A(\boldsymbol{b},\lambda)d\lambda \quad \text{(using } [A(\boldsymbol{b},\lambda)]^2 = 1) \tag{8}$$

Now $A(\boldsymbol{a},\lambda)A(\boldsymbol{b},\lambda) = \pm 1$, and $\rho(\lambda)[1 - A(\boldsymbol{b},\lambda)A(\boldsymbol{c},\lambda)] \geq 0$, so we can write:

$$\mathrm{I}E(\boldsymbol{a},\boldsymbol{b}) - E(\boldsymbol{a},\boldsymbol{c})\mathrm{I} \leq \int \rho(\lambda)[1 - A(\boldsymbol{b},\lambda)A(\boldsymbol{c},\lambda)]\,d\lambda \tag{9}$$

That we can rewrite to:

$$\boxed{\mathrm{I}E(\boldsymbol{a},\boldsymbol{b}) - E(\boldsymbol{a},\boldsymbol{c})\mathrm{I} \leq 1 + E(\boldsymbol{b},\boldsymbol{c})} \tag{10}$$

This is the famous *Bell-inequality*[6], and it is easy to show that it is in contradiction to quantum mechanics. Suppose **a**, **b** and **c** to be in a plane having 45° angles in between each others. Now, according to quantum mechanics (3), we have:

$E(\boldsymbol{a},\boldsymbol{b})^{qm} = -\boldsymbol{a}\cdot\boldsymbol{b} = cos\theta_{ab} = cos90° = 0.$ $E(\boldsymbol{a},\boldsymbol{c})^{qm} = E(\boldsymbol{b},\boldsymbol{c})^{qm} = -0.707.$

But if we put this result into Bell´s inequality, it is violated:

$$0.707 \nleq 1 - 0.707 = 0.293 \tag{11}$$

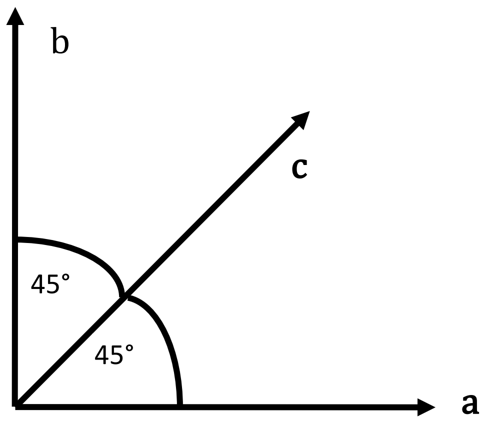

Figure 3: A possible detector orientation, violating the Bell inequality.

Bell tests

As we could see, the EPR critic of quantum mechanics became more precise through Bell´s work, and was set in a mathematical context. In the following decades, many Bell tests have been performed with an ever increasing preciseness. We should call into mind that up to that point, the discussion around the "completeness of quantum mechanics" or "correct description of reality" was based on *Gedanken experiments,* and first as late as in the 1970s, technology had caught up to test experimentally what so far just has been discussed theoretically.

An important step towards such a test was when in 1969 John Clauser, Michael Horne, Abner Shimony and Richard Holt[7] published a modification of Bell´s inequality that was better suited to real tests – the *Clauser-Horne-Shimoney-Holt- (CHSH) inequality*. It states, that all local hidden variable theories must obey:

$$\boxed{S(\boldsymbol{a},\boldsymbol{b},\boldsymbol{a}',\boldsymbol{b}') = \mathrm{I}E(\boldsymbol{a},\boldsymbol{b}) - E(\boldsymbol{a},\boldsymbol{b}')\mathrm{I} + \mathrm{I}E(\boldsymbol{a}',\boldsymbol{b}) + E(\boldsymbol{a}',\boldsymbol{b}')\mathrm{I} \leq 2} \quad (12)$$

Here, **a, b, a´, b´** are detector orientations. Now this result is, of course, as well violated by quantum mechanics. We archive the largest violation for the following orientation[VI]:

$$S(22.5°, 67.5°, 112.5°, 157.5°) =$$

$$= \mathrm{I}(-\cos 45°) - (-\cos 135°)\mathrm{I} + \mathrm{I}(-\cos 45°) + (-\cos 45°)\mathrm{I}$$

$$= \mathrm{I} - \tfrac{1}{2}\sqrt{2} - \tfrac{1}{2}\sqrt{2}\,\mathrm{I} + \mathrm{I} - \tfrac{1}{2}\sqrt{2} - \tfrac{1}{2}\sqrt{2}\mathrm{I} = 2\sqrt{2} \nleq 2 \quad (13)$$

In a Bell test, the expectation value is measured in the following way:

$$E(a,b) = \frac{N_{\uparrow\uparrow}(a,b) + N_{\downarrow\downarrow}(a,b) - N_{\uparrow\downarrow}(a,b) - N_{\downarrow\uparrow}(a,b)}{N_{total}} \quad (14)$$

$N_{\uparrow\uparrow}(\boldsymbol{a},\boldsymbol{b})$, for example, is the number of particle pairs with analyzer configuration **a, b** that have been measured to have spin $\uparrow_1\uparrow_2$. The other Ns are calculated respectively. N_{total} is the total number of particle pairs. We can easily see that $E(\boldsymbol{a},\boldsymbol{b})$ correctly takes the value "-1" for a single measurement with parallel detector orientations.

Already in the early 1970s, a first generation of Bell tests was performed, the first one in 1972 in Berkeley by Stuart J. Freedman and John F. Clauser[8]. These experiments were in accordance with quantum mechanics, but not very satisfying as they still contained some *loopholes*, meaning that not all possible local, realistic models as an explanation for the attained results could be excluded. There is, of course, in a strict sense no perfect experiment as there are always sources of error, but as the points of issue, locality and realism, were principles of a physical world view, it is worth to examine these *loopholes* in detail.

Locality Loophole

The most important *loophole* is the *locality loophole* which is sometimes also called *communication loophole*. It argues that a possible subluminal communication between the particles could be responsible for the correlation of the spins. Already earlier, we mentioned that Bell proposed that one could avoid the possibility of communication between two

[VI] The detector orientation refers to spin-½ particles. For photons which are spin 1 particles we have other spin-matrices, e.g.: $S_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$, leading to the expectation value $E(\boldsymbol{a},\boldsymbol{b})^{qm} = cos2\theta_{ab}$ and a maximum violation for the angles 0°,22.5°,45°,67.5° which usually are referred to as *Bell Testing Angles* due to the fact that most experiments use photons.

entangled particles by fixing the detector orientations, and thereby making the choice of which component of the spin is measured, just before the measurement.
More exactly, this means that we want the event "choice of detector orientation at A" (let´s call it "event A") to be space-like separated from the event "measuring spin at B" ("event B"), meaning in a space-time diagram that their future light cones do not cross.

However, in the experiment by Freedman and Clauser (as in the others of the 1st generation), the detector orientation was fixed in the instrumental setup at least hours before the measurement and the distance between A and B only several meters. The locality loophole was there-fore not closed at all. Furthermore, gave the use of single channel detectors (Fig. 4) which can only give access to one of the two outcomes ("up" or "down"), reason for critics to argue that not all photons passed through the polarizer.[9]

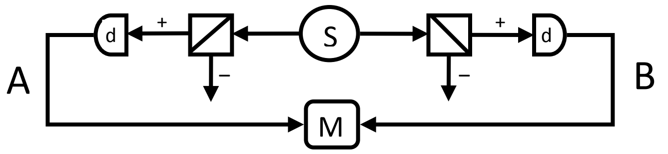

Figure 4: Experimental setup using single channel detectors. After leaving the source *S*, the particles are polarized but only one polarization can be detected by the detectors *d*, before the result becomes recorded in the monitor *M*.

Over the years, Bell tests became more complex and precise, and the *loopholes* were closed with an increasing certainty.

Alain Aspect´s test(s)
In 1982 a test with a setting much closer to Bell´s ideal was performed by Alain Aspect et al.[10] using double channel polarizer (Fig. 5), now being able to detect spin-"up"- and spin-"down".[VII] The experimenters also varied the detector orientation periodically after the photons had been emitted so that the possibility for a subluminal message exchange can be excluded. The distance between the detectors was 12m what takes light 40nsec to travel for.

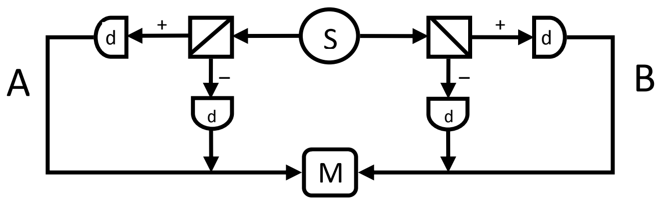

Figure 5: Experimental setup using double channel polarizer. Both spin up and spin down particles are detected and recorded.

For his experiment, Aspect built smart "switching devices" which should choose the component of the spin to be measured at A and B (Fig. 6). A transducer driven in phase produces standing waves in a slab of water with about 25MHz (slightly different frequency at A than at B), through which the photons must pass. The light falls in in Bragg-angle θ_B, and either is transmitted in the case of the amplitude of the standing wave being zero, or deflected in angel $2\theta_B$ for the case that the amplitude is maximum. Photons hitting the water in an intermediate phase are deflected in another angle, and are thus not detected.

[VII] There actually was another, earlier experiment form A. Aspect et al. that often is quoted as a milestone in Bell tests, but we look closer at the chosen one as it suits better for a short presentation. The first one was: Aspect, A., Grangier, P., & Roger, G. (1982*)*. Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell´s Inequalities. *Physical Review Letters 49*, 91–94.

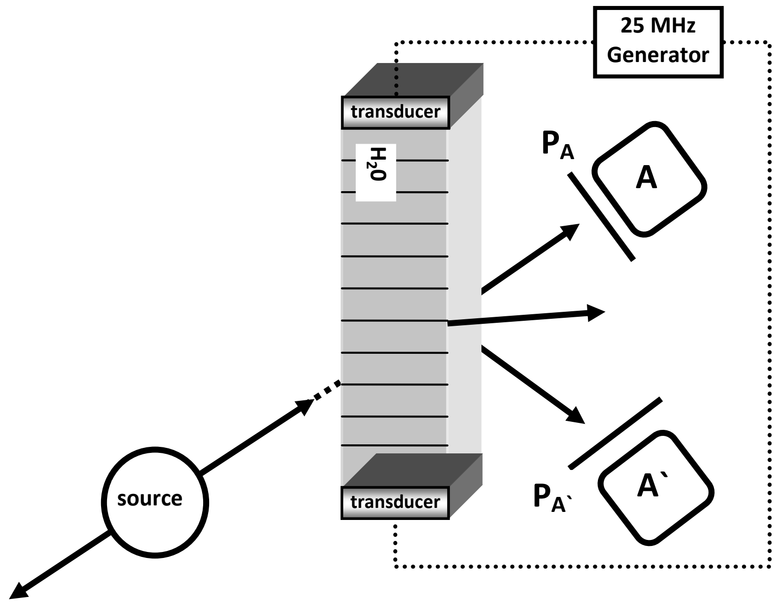

Figure 6: Schematic sketch of the „switching-device" that was used by Alain Aspect. A photon hits a water slap in a certain phase of a standing wave produced by the 25MHz generators. The standing wave in the water slap acts like a diffraction grating. If the photon hits the wave at a node, no deflection takes place. If the photon however hits the wave at an anti-node, it is deflected in double Bragg-angel. Photons hitting the wave in an intermediate phase are deflected in different angles where no detector is set up.

A frequency of 25MHz corresponds to 40ns for a full period, in which we have 4 changes from node to anti node. This means that the resulting switching time of 10ns between the detector orientations A and A´ is relatively small in comparison to the 40ns it would take for a luminal message to get from the switching device at A to the one at B, and to transmit the information of which component is measured at A. "Event A" and "event B" are therefore truly space-like separated. In Aspect´s experiment, $S(\boldsymbol{a},\boldsymbol{b},\boldsymbol{a}',\boldsymbol{b}')$ was found to be 2.697 ± 0.015. The CSHS inequality (12) was clearly violated and quantum mechanics therefore approved.

Still some critics of quantum mechanics did not see the *locality loophole* fully closed. They questioned if the switching in fact was a truly random process, what it would need to be in an ideal Bell test. We remember that the generators at A and B are driven in different frequencies so that there is no correlation between the choices, and as shown before, there would also be no time for an information to travel between the switching-devices. The point critics attacked was that since the devices are driven periodically, there still is theoretically the possibility that the switching station at B can (with the help of sub-luminal information exchange) predict what the setting at the station at A will be at a certain time and vice versa.[11]

The Innsbruck experiment of truly random choice

Nevertheless, also the problem of a truly random detector orientation choice while the particles are already in flight, was solved over time. In 1998 Gregor Weihs et al.[12] made an experiment at the University of Innsbruck, in which they used a *physical random number generator* (RNG), sometimes also called *true random number generators,* to determine the detector orientation at each of the two detection stations.

Such a generator consists of a light-emitting diode with a coherence time of $t_c \approx 10fs$, meaning it emits nearly completely polarized light. This diode is directed on a beam splitter. The resulting two beams are each detected by a photomultiplier which transforms the incoming photons into an amplified, electrical signal. Depending on which photomultiplier detects a photon, let us call them "0" and "1", the final output of the RNG than is "0"/"1". The attained number is the completely random choice for if the spin is measured in **a** or **a´** direction. If both photomultipliers are hidden by a photon within 2ns, these detections are ignored. The maximum switching frequency is therefore 500MHz. The total time for a switch between two detector orientations in the used device, adding all delays of its consisting parts together, was measured with 75ns. Adding another 25ns for additional possible effects, one obtains a switching time of maximum 100ns. This is much shorter than the $1.3\mu s$ that a signal with the speed of light would need to travel the 400 meters that the two detection stations were separated in the experiment.
What is more was the Innsbruck experiment performed in a way carefully excluding any common context in the registration. In all previous experiments have the measurement results been registered by a common monitor (Fig. 4/5). Now, for the first time, the data were registered separately at A and B, and due to the shorter signal time the whole registration process well completed within$1.3\mu s$.
The two measurement events at A and B, including the registration of the results, were therefore surely space-like separated, and recorded by two completely independent detecting stations. In this experiment, 14,700 particles were detected, and a $S(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{a}', \boldsymbol{b}')$-value of 2.73 ± 0.02 was achieved.

The Innsbruck experiment erased the last hopes for local hidden variable theories. Einstein´s view, that the watched correlations of entangled particles were already determined in the common source, and that the information about them was carried by each particle all along the way of up to its detection, could now surely not be held any longer. An entangled photon pair must be seen as a non-separable object. In fact, all experiments point to that the correlation between the pair of particles happens instantaneously as the Copenhagen interpretation postulates.
At the same time, we should remind ourselves that entanglement does not provide a possibility to send messages from one detection station to another with a speed faster than light. There is not way to influence the outcome of an experiment at one detection station, so that sequences of measurement results as a code for messages could be sent from A to B. Entanglement merely constitutes an instantaneous correlation of otherwise completely random experimental outcomes.

Other loopholes

Detection efficiency loophole

Another often named loophole is the *detection-efficiency loophole* which criticizes that the detected particles do not represent a fair sample of all the particles that were emitted by the source. In fact, are in most of the experiments (especially in all the ones that use photons) only a small subset of all created particles also detected. We can only assume that they represent a fair sample, what would be statistically reasonable and also is assumed by most scientists. This assumption is often referred to as the *fair-sampling assumption*.

In addition, the *detection-efficiency loophole* was finally closed in 2001 by David Wineland et al.[13]. They performed a Bell test with massive entangled particles, ${}^{9}_{4}Be^{+}$-ions, and results were found to be in accordance with quantum mechanics.

The advantage of ions compared to photons is that they can be detected with efficiency close to 100%.

Yet did this experiment not close the *locality loophole*. The entangled Be^{+}-ions were only separated approximately 3μm, and although no know interaction could have affected the results, the measurement events "A" and "B" were not space-like separated.

In 2008 Dzmitry Matsukevich et al.[14] attained improved results using ${}^{171}_{70}Yb^{+}$-ions (Ytterbium) that were separated 1meter, but for surely clothing the locality loophole a distance of as much as 15km would be required. From the present stand of technique it is unlikely that such long optical ion traps can be built in the near future.[15]

Spatial correlation loophole

As a last loophole one can mention the *spatial correlation loophole* which argues that if we have a particle source where a third particle is involved, for example a two photon cascade in an atom, the third one ,here the atom, could absorb some of the momentum. Than the two entangled particles would not have the strong spatial correlation that is needed to be sure that both particles end up in the detector. In fact, two photon atomic cascades were used for the first Bell tests, as for example in the one named above performed by Freedman and Clauser. Though, this loophole was easily closed by using two-body processes, especially by the use of *parametric down conversion*. In this procedure, a photon of high energy is split with the help of a nonlinear crystal in a pair of two photons while momentum and energy are conserved.

Preferred particles for a Bell test

In the various Bell tests performed, a range of different particles were used; protons, neutrons, K-mesons, B-mesons, photons, ions and even atoms.

As we have seen, ions are the best particles for clothing the *detection-efficiency loophole*, but the difficulty to separate them in large distances makes them unattractive for most tests so far.

Photons have the advantage that they can be fed in optical fibers and be separated in large distances.[VIII] This property, combined with their high velocity makes photos the best particle with which the *locality loophole* can be closed. As this is the most fundamental *loophole*, most experiments were and are done using entangled pairs of photons. On the other hand, the detection efficiency is despite much effort with about 30% as top values[16] still relatively low, and has yet to be improved for attaining a perfect Bell test that closes all loopholes at the same time.[17]

If not instantaneous, than at least 10,000 times faster than light

According to quantum mechanics, correlations between particle pairs happen instantaneously, and without any information traveling between particles. Experimentally, however, that is

[VIII] E.g.: More than 10km distance was achieved in: Tittel, W., Brendel, J., Zbinden, H. & Gisin, N. (1998). Violation of Bell inequalities by photons more than 10km apart. *Physical Review Letters*, 3563-3566.

difficult to prove, and critics of the Copenhagen interpretation have argued for a communication at finite speed. In 2008 Daniel Salart et al.[18] published the results of an experiment, in which they pretended to have proven that the minimum speed needed for a possible communication between entangled particles is 10,000 times faster than the speed of light.

In all Bell tests performed so far, the measurement events A and B have never been 100% simultaneously due to technical imperfection, but, theoretically, there exists a preferred reference frame in which they are.

To understand this, we turn to a famous *Gedanken experiment* of the theory of special relativity: Consider a wagon traveling at constant speed v along a straight track (Fig. 7). Now a bulb that hangs exactly in the middle of the wagon is switched on. For an observer in the wagon, the light reaches the ends of the wagon exactly at the same time. In other words, the events "X" – light reaches back end – and "Y" – light reaches front end – are completely simultaneous in a reference frame co-moving to the wagon.

Yet for an observer on the ground, these two events are not simultaneously. The light still is emitted completely in the middle between "E" and "F", but as both ends are moving to the right, for an observer on the ground, the light has a shorter way to "E" than to "F". As the speed of light is the same in all inertial reference frames, event "X" occurs before event "Y" in the resting frame.[19]

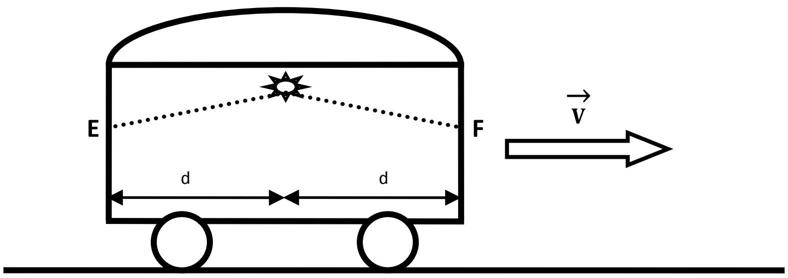

Figure 7: Sketch of the *Gedanken experiment* about simultaneity depending on the reference frame. A light source in middle of a wagon which moves with velocity v along a straight track is switched on. In a co-moving frame the light seems to hit the front and back end simultaneously whereas it seems to reach the back end first in a resting frame.

Analog to this example, we can imagine that there is a certain preferred reference frame in which the measurements at two entangled particles are perfectly simultaneous.

In this case, there would be no time for any information exchange between our detection stations at A and B, even if the signal was a billion times faster than light. Therefore, if entanglement should be caused by communication between the particles at finite speed, no entanglement should be watched. Salart and colleagues wanted to test that by performing a Bell test over more than 24 hours.

The idea

If the events "A" and "B" have a certain alignment/time-difference in the earth frame, then, according to special relativity, they have exactly the same alignment in any reference frame moving perpendicular to the A-B axis.

If now the AB-axis in our experiment is east-west oriented, then all possible privileged reference frames would be scanned within 12 hours (Fig. 8). At a certain point of time during these 12 hours, the preferred reference frame would have the same local time of the events "A" and "B" than the Earth frame, and therefore the speed of any communication between A and B have the same bounds.

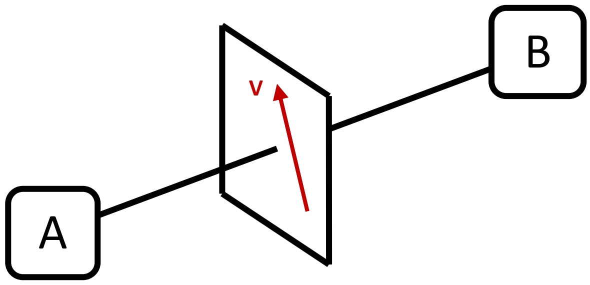

Figure 8: Illustration of the setup that scans all possible reference frames. Imagining the plane perpendicular to the A-B axis rotating 180° within 12h one can nicely see that all possible directions in which the preferred reference frame could move are included.

We now assume an inertial frame centered on the Earth in which the measurement events "A" and "B" occur at t_A and t_B at positions r_A and r_B. In the inertial[IX], preferred reference frame F, moving with velocity **v** in respect to the Earth frame, the speed of a communication between A and B would be:

$$V_{QI\ in\ F} \geq \frac{\mathrm{I}\, r'_B - r'_A\, \mathrm{I}}{\mathrm{I}\, t'_B - t'_A\, \mathrm{I}} \tag{15}$$

with (r'_A, t'_A) and (r'_B, t'_B) being the coordinates of event A and B in F due to Lorentz-transformation. "QI" stands for *Quantum Information*, the supposed communication between the particles. Equation (15) we can transform into:

$$\left(\frac{V_{QI\ in\ F}}{c}\right)^2 \geq 1 + \frac{(1-\beta^2)(1-\rho^2)}{(\rho+\beta_{\parallel})^2} \tag{16}$$

Here, $\beta = v/c$ is the speed of the Earth frame in frame F relative to the speed of light, $\beta_{\parallel} = v_{\parallel}/c$ with $v_{\parallel}$ is the part of **v** parallel to the A-B axis, and $\rho = c\frac{t_{AB}}{r_{AB}}$ is the alignment of the two measurement events on the Earth frame.

For the experiment, real space-like separated events were assumed, that means $\rho \leq 1$.

Finding a lower bound for $V_{QI\ in\ F}$

The experimenters now had to find a lower bound for $V_{QI\ in\ F}$ to give a real lower bound for the speed of quantum information. To do that, they had to bound ρ and $\beta_{\parallel}$ from above.

In principle ρ would have to be optimized for each privileged frame, but here was assumed that $\mathrm{I}\rho\mathrm{I} \leq \rho` \ll 1$ where ρ\` is maximum uncertainty of the experimental setup to perfect simultaneity. That changes equation (16) to:

$$\left(\frac{V_{QI\ in\ F}}{c}\right)^2 \geq 1 + \frac{(1-\beta^2)(1-(\rho`)^2)}{(\rho`+\beta_{\parallel})^2} \tag{17}$$

[IX] Note by the author: in an accelerated frame, we would need to calculate the effects of the acceleration according to the theory of general relativity using the Einstein-Equivalence-Principle which says that there is no way in which we could locally differ between the effects of an accelerated frame, and the effects of a homogenous gravitational field.

To find an upper bound of $\beta_{\|}$ we take a look at Figure 9. Here, α is the angle between the x-y plane and the line through A and B. Consider now a unit vector on the A-B axis. It has a fixed z-component, and its component in the x-y plane rotates with angular velocity ω.

$$\boldsymbol{e}_{AB}(t) = sin\alpha\boldsymbol{e}_z + cos\alpha\boldsymbol{e}_{xy}(t)$$

Analogously, we can split up the velocity $\boldsymbol{v}$ in terms of $\boldsymbol{\beta} = \boldsymbol{v}/c$:

$$\boldsymbol{\beta} = \beta cos\chi\boldsymbol{e}_z + \beta sin\chi\boldsymbol{e}_{xy}^F$$

Now :

$\beta_{\|}(t) = \boldsymbol{\beta}\boldsymbol{e}_{AB}(t)$

$= \beta cos\chi sin\alpha + \beta sin\chi cos\alpha\boldsymbol{e}_{xy}^F\boldsymbol{e}_{xy}(t) =$

$= \beta cos\chi sin\alpha + \beta sin\chi cos\alpha cos\omega t \qquad (18)$

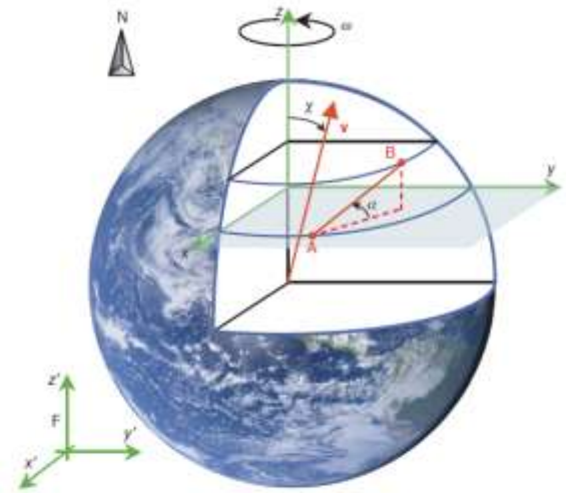

Figure 9: **Reference frames. The Earth frame moves with respect to a hypothetically privileged reference frame F at a speed v. The zenith angle χ between v and the z-axis can have values between 0° and 180°. The A-B axis forms an angle α with the equatorial (x–y) plane. ω is the angular velocity of the Earth. Figure copied form Salart´s article18.**

with $\boldsymbol{e}_{xy}^F\boldsymbol{e}_{xy}(t) = cos\omega t$. That the latter term is of sinusoidal nature becomes clear, if one imagines the coordinate system of the earth frame rotating in the F frame. If one writes it as sinus or cosines depends on where one chooses the origin of time.
As explained before, there will be a point of time t_0 in which **v** is perpendicular to the A-B axis where $\beta_{\|}(t_0) = 0$. The bound for $\beta_{\|}$ is obtained by bounding it for a small time interval T around t_0 with the upper formula and thus obtain a high lower bound for $V_{QI\ in\ F}$. The angle α is fixed by the experimental setup, and χ stays variable as it depends on the direction of **v.**

Experimental setup

The two detection stations were located in the villages of Satigny and Jussy, and were separated by a direct distance of $r_{AB} = 18.0$ km (Fig. 1). The A-B axis had an imperfection to a real east-west orientation of $\alpha = 5.8°$. The fibre length from the source to the stations was each 17.5 km and the measurement events "A" and "B" therefore theoretically simultaneously. Nevertheless, the experimental setup contained uncertainties of all together up to $t_{AB} \leq 323$ps. These values cause an alignment of $|\rho| \leq \rho` = 5.4 \cdot 10^{-6} \ll 1$. The average time to observe a Bell violation was T=360s. $V_{QI\ in\ F}$ could now be calculated as a function of χ and β. It was assumed that $\beta \leq 10^{-3}$, meaning that the

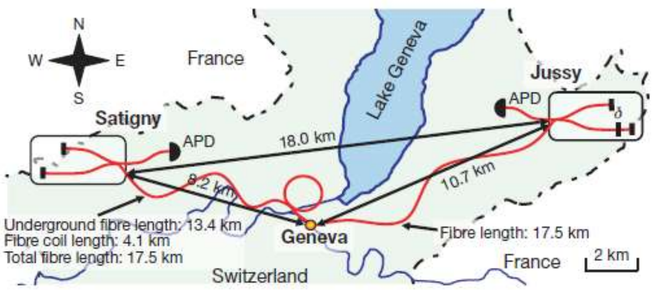

Figure 10: **Experimental setup. The source sends pairs of photons from Geneva to two receiving stations through the Swisscom fibre-optic network. The stations are situated in two villages, Satigny and Jussy, that are respectively 8.2 and 10.7km from Geneva. The direct distance between them is 18.0 km. Figure copied form Salart´s article[18].**

relative speed between F and the Earth frame was set to be maximum 3,000 km/s. With that assumption, the lower bound of $V_{QI\ in\ F}(\chi)$ was found to be 10,000 times the speed of light.

Critiques to Salart´s Bell test.

As astonishing as the result may be, Salart`s test received critics for not closing the *locality loophole* properly[X]. The “setting choice”-event at Satigny was not space-like separated from the “detection”-event in Jussy, as the instrumental setup at Satigny was stationary.

Furthermore, it was criticized that the used experimental setup does not fit the *Clauser-Horne-Shimony-Holt-Bell inequality* which was applied in the experiment. Even with a truly random choice, experimental setup and used inequality would have been incompatible[20].

Hidden-variable theories

As we have seen, quantum mechanics violates the Bell inequality which is obeyed by all local hidden variable theories. Various Bell tests approved quantum mechanics and opposed local hidden variable theories. That view is moreover strengthened by another theorem, the so-called *Kochen-Specker theorem*, discovered by Simon Kochen and Ernst Specker in 1967[21]. The theorem states that there is no *contextual* hidden variable theory. *Contextuality* means in that case that if a quantum mechanical system is measured to have a certain property, it has this property independent to the way the measurement was made, and also independent to the measurement apparatus. The locality condition which sees the measurement at A as independent of the measurement at B is hereby included and the *Kochen-Specker theorem* therefore another disproof for local hidden variable theories. Actually, Kochen and Specker had built their work on that of John von Neumann, who already in 1932 argued for the incompatibility of quantum mechanics and hidden variable theories, but had made some wrong assumptions[22].

As we have seen, one can not hold the idea that all of nature obeys the principle of locality. Still there remains the possibility of non-local hidden variable theories that allow a realist world view. The best known of them is the *Bohm interpretation* of quantum mechanics proposed by David Bohm in 1952[23], which is sometimes also called *Casual interpretation, De Broglie-Bohm interpretation or pilot wave theory*. In Bohm´s interpretation, each particle has an individual history evolving in space time. Just as a classical object, it follows a determinable trajectory in space.

Bohm in fact just reinterpreted Schrödinger´s *wave formalism* in a way that preserves a realist world view. A similar attempt was already made in 1926 by Louis de Broglie,[24] but given up later again as it was criticized intensely, and the success of the Copenhagen interpretation seemed to verify the critics.

Just as all other critics of quantum mechanics, Bohm did not like the idea that the wave function represents the most complete description of a quantum system, and that it gives only statistical predictions to a measurement outcome as a property of nature itself. Bohm

---

[X] For more details see: Kofler J., Ursin, R., Brukner, C., Zeilinger, A. *Comment* on: Testing the speed of ‘spooky action at a distance; available in the internet e.g. on http://en.scientificcommons.org/38059590.

explained his motivation was to find an alternative interpretation that "permits us to conceive of each individual system as being in a precisely definable state, whose changes with time are determined by definite laws, analogous to (but not identical) with the classical equations of motion. [...] Quantum mechanical probabilities are regarded (like their counterparts in classical statistical physics) as only a practical necessity, and not as a manifestation of an inherent lack of complete determination in the properties of matter at the quantum level."[25]. Bohm reminded to the discovery of the atomic theory. At that time, certain effects, as the Gas Law, inspired some scientists to postulate the existence of atoms. Yet as these effects could also be explained by the already existing theory of thermodynamics, many doubted the atomic theory. First after some time, atoms and their behavior were proven to be the underlying principle and in Bohm´s eyes, it could be the same with quantum mechanics.
To find such underlying causes, Bohm argued for a new interpretation of the Schrödinger equation. As he did not change the general form of the Schrödinger equation, his interpretation leads to the same predictions as the Copenhagen one in accordance with experimental results.

New interpretation of the Schrödinger Equation

Bohm substituted the wave function $\Psi = Re^{iS/\hbar}$ [XI] into the Schrödinger equation which states:

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r},t)\right)\Psi(\boldsymbol{r},t) \qquad (19)$$

By substitution we obtain:

$$\left[i\hbar\partial_t + \frac{\hbar^2}{2m}\nabla^2 - V(\boldsymbol{r},t)\right]Re^{\frac{iS}{\hbar}} = 0$$

$$= i\hbar(\partial_t R)e^{\frac{iS}{\hbar}} + i\hbar \mathrm{R}e^{\frac{iS}{\hbar}}\frac{i}{\hbar}\partial_t S + \frac{\hbar^2}{2m}\nabla\left(\nabla \mathrm{R}e^{\frac{iS}{\hbar}} + Re^{\frac{iS}{\hbar}}\frac{i}{\hbar}\nabla \mathrm{S}\right) - V(\boldsymbol{r},t)Re^{\frac{iS}{\hbar}}$$

$$= i\hbar e^{\frac{iS}{\hbar}}\left(\partial_t R + R\frac{i}{\hbar}\partial_t S\right) + \frac{\hbar^2}{2m}e^{\frac{iS}{\hbar}}\left[\nabla^2 R + 2\frac{i}{\hbar}\nabla \mathrm{R}\nabla \mathrm{S} + \mathrm{R}\left(\frac{i}{\hbar}\nabla \mathrm{S}\right)^2 + R\frac{i}{\hbar}\nabla^2 \mathrm{S}\right] -$$

$$-V(\boldsymbol{r},t)Re^{\frac{iS}{\hbar}} = 0 \qquad (20)$$

Dividing by $e^{\frac{iS}{\hbar}}$ we obtain a sum of imaginary and real elements. For that equation to be zero, the sum of all imaginary elements must be zero as well as the sum of all real elements. Using that $i^2 = -1$, we obtain the two following equations:

Real part:

$$-\mathrm{R}\partial_t S - \frac{\mathrm{R}}{2m}\nabla^2 \mathrm{S} + \frac{\hbar^2}{2m}\nabla^2 R - V(\boldsymbol{r},t)R = 0$$

[XI] R(**r**, t) is controlling the amplitude and S(**r**, t) is the phase of the wave. Any solution of the Schrödinger equation can be expressed in this form.

$$\boxed{\partial_t S + \frac{\nabla^2 \mathrm{S}}{2m} - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{\mathrm{R}} + V(\boldsymbol{r}, t) = 0} \qquad (21)$$

Imaginary part:

$$i\hbar(\partial_t R) + \frac{i\hbar}{m} \nabla \mathrm{R} \nabla \mathrm{S} + \frac{i\hbar}{2m} \mathrm{R} \nabla^2 \mathrm{S} = 0 \qquad (22)$$

We now modify equation (22) by dividing through $i\hbar$ and remember that $P = \mathrm{I}\Psi(\boldsymbol{r}, t)\mathrm{I}^2 = \mathrm{R}^2$, leading to $\mathrm{R} = (\pm)\sqrt{\mathrm{P}}$, $\partial_t \sqrt{\mathrm{P}} = \frac{1}{2\sqrt{\mathrm{P}}} \partial_t P$ and $\nabla \sqrt{\mathrm{P}} = \frac{1}{2\sqrt{\mathrm{P}}} \nabla P$. Thus we obtain:

$$i\hbar\left(\frac{1}{2\sqrt{\mathrm{P}}} \partial_t P\right) + \frac{i\hbar}{m}\frac{1}{2\sqrt{\mathrm{P}}} \nabla P \nabla \mathrm{S} + \frac{i\hbar}{2m}\sqrt{\mathrm{P}} \nabla^2 \mathrm{S} = 0$$

we multiply with $2\sqrt{\mathrm{P}}$ and rewrite $P = \mathrm{I}\Psi(\boldsymbol{r}, t)\mathrm{I}^2 = \mathrm{R}^2$:

$$i\hbar(\partial_t \mathrm{R}^2) + \frac{i\hbar}{m} \nabla \mathrm{R}^2 \nabla \mathrm{S} + \frac{i\hbar}{m} \mathrm{R}^2 \nabla^2 \mathrm{S} =$$

$$= \boxed{\partial_t R^2 + \nabla\left(R^2 \frac{\nabla \mathrm{S}}{m}\right) = 0} \qquad (23)$$

Interpretation of imaginary part – the continuity equation

When we replace R^2 with P, equation (23) looks similar to the continuity equation.

$$\partial_t P + \nabla\left(P \frac{\nabla \mathrm{S}}{m}\right) = 0 \qquad (24)$$

$$\partial_t P + \nabla J = 0 \qquad \textit{Continuity equation} \qquad (25)$$

And in fact when we calculate J for the given wave function $\Psi = Re^{iS/\hbar}$, we see that the equation of the imaginary part is the continuity equation for quantum mechanics, and is a term for probability conservation.

$$J = \frac{\mathrm{i}\hbar}{2m}(\Psi \nabla \Psi^* - \Psi^* \nabla \Psi) = \frac{\hbar \mathrm{i}}{2m}\left(Re^{iS/\hbar} \nabla Re^{-iS/\hbar} - Re^{-iS/\hbar} \nabla Re^{iS/\hbar}\right)$$

$$= \frac{\hbar \mathrm{i}}{2m}\left[\mathrm{R}e^{\frac{iS}{\hbar}}\left(\nabla \mathrm{R} - \mathrm{R}\frac{i}{\hbar} \nabla \mathrm{S}\right) e^{-\frac{iS}{\hbar}} - \mathrm{R}e^{-\frac{iS}{\hbar}}\left(\nabla \mathrm{R} + \mathrm{R}\frac{i}{\hbar} \nabla \mathrm{S}\right) e^{\frac{iS}{\hbar}}\right]$$

$$= \frac{\hbar \mathrm{i}^2}{2m}\left[-2\mathrm{R}^2 \frac{i}{\hbar} \nabla \mathrm{S}\right] = R^2 \frac{\nabla \mathrm{S}}{m} \qquad (26)$$

Interpretation of the real part – quantum potential and guiding wave

Equation 21 actually only differs by one term from the classical Hamilton-Jacobi equation which describes the mechanical state of a system.

$$\boxed{\partial_t S + \frac{\nabla^2 S}{2m} - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} + V(\boldsymbol{r},t) = 0} \tag{21}$$

$$\frac{\partial}{\partial t}S_c + \frac{(\nabla S_c)^2}{2m} + V = 0 \qquad \textit{Hamilton Jacobi Equation} \tag{27}$$

Comparing equations (21) and (27), we can see that the quantum mechanical equation has an extra term, and that the classical action function S_c[XII] is replaced by the phase-term of the wave equation. The extra term:

$$Q \equiv -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{28}$$

is called the *quantum potential*[XIII]. Bohm assumed that the classical relations $\boldsymbol{p} = \nabla S_c$ and $E = -\partial_t S_c$ are also valid in quantum mechanics, and replaced S_c with S. In the classical limit, when $\hbar = 0$, the quantum potential disappears, and we remain with the Hamilton-Jacobi equation. Bohr argued now that for the non-classical case of $\hbar \neq 0$, the extra term should be seen as an extra potential, and equation 21 therefore as a *quantum Hamilton-Jacobi equation* providing a set of trajectories for the quantum particle.

The extra potential depends on R and therefore on $\Psi(\boldsymbol{r},t)$. For Bohm that gave rise to regard the wave function as a mathematical representation of an objectively real field that exerts a force on a particle similar to an electromagnetic field. This field is for Bohm a sort of guiding wave, precisely determining the particle´s position and momentum at each point of time.

As the Hamilton-Jacobi equation is just one possibility to calculate the equation of motion, Bohm suggests that one could just as well modify Newton´s equation of motion to the quantum mechanical case by adding the *quantum potential*:

$$m\frac{d}{dt}\boldsymbol{v} = -\nabla(V + Q) \tag{29}$$

Explaining the EPR-Paradox in terms of a Ψ-field

An important application of Bohm´s supposed Ψ-field is to explain entanglement with it. According to Bohm, the two-particle system has a six-dimensional wave field which would undergo uncontrollable fluctuations in case of a measurement. If then the momentum of one particle is measured, the Ψ-field of the whole system transmits the thereby happening disturbance instantaneously to the other particle.

[XII] According to classical mechanics, an object follows that path for which the action is minimized. This is called "the principle of least action". With its help, the classical equations of motion for the object can be derived. The action function is the integral over the Langrangian, $S_c = \int_{t_2}^{t_1} L(q, \frac{\partial}{\partial t}q, t)\, dt$, where the Langrangian describes the mechanical state of a system using the generalized coordinates q_i, its derivatives (velocity) and time. Expressed in Cartesian coordinates the Langrangian is $L = \frac{1}{2}mv^2 - V(r)$

[XIII] The term „potential" which seems natural when comparing equations (26) and (27) is misleading as the term has nothing to do with a classical potential.

In addition, the Ψ-field should help to understand the double-slit experiment without the two concepts of photons as particles and waves depending on what we measure, but with the single concept of a particle with a guiding wave explaining the outcome of all experiments.

The uncertainty principle

Although Bohm´s interpretation leads to an equation that allows calculating a set of trajectories, it still is not possible to know p and x of a particle simultaneously with an arbitrary preciseness. Bohm´s interpretation does not violate the uncertainty principle which forbids us exactly that. A violation is also not expectable as Bohm did not change the wave-formulation, but only reinterpreted it. However, the uncertainty principle does not provide any information to the question if the particle really does not have a simultaneous position and momentum or if it does, but it is simply impossible to measure it. The latter position is the one favored by Bohm.

Modification of Schrödinger´s Equation

What is more, Bohm promoted his interpretation by punctuating that it allows a mathematical modification of the Schrödinger equation which could help to explain phenomena in the domain of distances of less than 10^{-13}cm. At the time of Bohm´s publication, phenomena on that scale could not be explained by quantum mechanics.
Bohm suggested to modify his version of the Schrödinger equation with an extra term that is only needed to explain the phenomena on the scale under 10^{-13}cm. The unmodified Schrödinger equation is then just seen as a good approximation on a larger scale.
In the Copenhagen interpretation, a modification of the Schrödinger equation is not possible as it is seen as the most complete description of the system. However such a modification was not needed to develop elementary particle physics which explains phenomena on the scale under 10^{-13}cm, and was developed on the base of quantum mechanics.

With his suggested interpretation of quantum mechanics in terms of hidden variables, Bohm wanted to show that it is possible to stick to a realist worldview and still obtain explanations for observed phenomena. Bohm did not in detail discuss all elements of quantum mechanics, but by explaining a few, he encouraged for further research on quantum mechanics in terms of hidden variables.
Non-local hidden variable theories were not disproven over the time, yet its ideas were not followed by many scientists after Bohm. It was as late as 1993 that the first comprehensive book about a "quantum theory of motion" appeared.[26]

Technical applications of entanglement

Entanglement can not only be used to experimentally prove non-locality, it also has technical applications. The research for such a use of quantum mechanical phenomena is called *Quantum information science* and the most developed application within it is *quantum cryptography*.
With the help of *quantum cryptography,* messages can be send with 100% security that a possible eavesdropper would be detected. The principle is rather simple: One measures entangled pairs of particles having parallel oriented detectors at the detection stations. On

both sides, one obtains a completely random, but correlated sequence of measurement results. This sequence can be used as a key to encrypt messages. This key is safe because of its total randomness. A possible eavesdropper could be detected as he would disturb a particle´s state by any measurement on it. The observed data at our detection stations then would not violate the Bell Inequality any more.

Another possible but still much less developed application is *quantum computing*. In *quantum computing,* one wants to use quantum properties (i.e. entangled states) to represent data – so called quantum bits (qbits). The advantage of such a quantum computer would be a decisively higher computing power compared to ordinary once.[27]

Conclusion

With the help of Bell´s inequality and the later on following tests, one of the most severe disputes within the scientific society was solved.

In the EPR-paper, critics of quantum mechanics had thought to have found a weak point of the theory that finally will lead to the preservation of the classical physical worldview as the only model to describe nature.

The solution to the critics came not abruptly, as we saw, but was the result of decades of research. The first milestone was John Bell´s inequality which provided the possibility for experimental tests. The Bell tests proved to require an exceedingly high technological standard to give reliable results. Despite many difficulties, such advanced tests have been developed and *loopholes* of *locality, spatial correlation* and *detection efficiency* have been closed. Nevertheless, a Bell test that closes all the loopholes at the same time still has to be waited for, and that even recent test as the one by Salart et al. not closed the locality loophole in a strict sense, shows how carefully tests involving entangled particles have to be performed.

However, through Bell tests the EPR-critic finally was solved. Nature was proven to be non-local on the quantum level. The dispute showed once more that scientists basically always must be open to change their model of the world. Also basic physical principles are only true as long as they are not disproven. Although all discoveries in science before quantum mechanics pointed to that all of nature behaves in a deterministic way, it is not the case. There is also no problem in it. Nature is both, determinate on the large scale, and indeterminate on the very small scale. The intensity with which the dispute about the completeness of quantum mechanics was held arose from the difficulties some scientists had with accepting this.

Quantum mechanics did not rule out an old model of the world as wrong, it just supplemented it. And hereby it was very successful. It was quantum mechanics that made the development of modern electronics possible. It would have rarely come to an information age without the technical developments arisen out of a better understanding of phenomena on the quantum level.

It will be interesting to see what happens with the alternative interpretations of quantum mechanics that not have been sorted out so far. Will there be a peaceful coexistence where at some point the interest in alternative interpretations as the *Bohm interpretation* could rise again? We do not know. But for sure we should not think that people working on them waste their time only because the Copenhagen interpretation works so nicely. It was John Bell´s occupation with local hidden variable theories that ultimately led to the proof of nature being non-local.

Finally, it is also worth to consider that entanglement added a new dimension to our idea of space. With the *Theory of Relativity*, we found out that space and time are not absolute, but depend on local conditions. Now, quantum mechanics sees a pair of entangled particle as a non-separable object. As long as there are no disturbances, each of the particles stays in a superposition of two states, and when they are measured, the correlation happens instantaneously no matter how large the distance between the particles might be. That means we can have an immediate knowledge about something that is in a classical sense totally separated from us, but at the same time we can not influence it.

Literature:

Books:

Bell, J.S. (1987). *Speakable and Unspeakable in Quantum Mechanics*. Cambridge: Cambridge University Press.

Bertlmann, R. A., Zeilinger, A. (2002). *Quantum [Un]Speakables – From Bell to Quantum Information*. Berlin: Springer Verlag.

Greenberger, D, Hentschel, K., Weinert, F. (2009). *Compendium of Quantum Physics*. Berlin: Springer Verlag.

Griffiths, D.J. (1999). *Introduction to Electrodynamics.* Upper Saddle River, New Jersey: Pearson Education.

Griffiths, D.J. (2005). *Introduction to Quantum Mechanics*. Upper Saddle River, New Jersey: Pearson Education.

Holland, P. R. (1993). *The Quantum Theory of Motion – An Account of the de Broglie-Bohm Casual Interpretation of Quantum Mechanics*. Cambridge: Cambridge University Press.

Le Bellac, M. (2006). *Quantum Physics*. Cambridge: Cambridge University Press.

Sakurai, J.J. (1994). *Modern Quantum Mechanics.* Reading, Massachusetts: Addison-Wesley Publishing Company.

Schommers, W. (1989). *Quantum Theory and Pictures of Reality*. Berlin: Springer Verlag.

Articles:

Aerts, S., Kwiat, P., Larsson, J. A., & Żukowski, M. (1999). Two-photon Franson-type experiments and local realism. *Physical Review Letters 83*, 2872-2876.

Aspect, A. , Dalibard, J. & Roger, G. (1982). Experimental Test of Bell's Inequalities Using Time – Varying Analyzers. *Physical Review Letters 49*, 1804-1807.

Bell, J.S. (1964). On the Einstein Podolsky Rosen Paradox. *Physics 1*, 195-200.

Bohm, D. (1952). A Suggested Interpretation of the Quantum Theory in Terms of `Hidden´ Variables I and II. *Physical Review 85*, 166-193.

Bohm, D. & Ahronov, Y. (1957). Discussion of Experimental Proof for the Paradox of Einstein, Rosen and Podolsky. *Physical Review 108,* 1070 - 1076.

Clauser, J.F.,Horne, M.A., Shimony, A. & Holt, R.A. (1969). Proposed experiment to test local hidden variable theories. *Physical Review Letters 23,* 880-884.

De Broglie, L. (1926). Sur la possibilité de relier les phénomènes d'interférences et de diffraction à la théorie des quanta de lumière. *Comptes Rendus 183*, 447-448.

Einstein, A., Podolsky, B. & Rosen, N. (1935). Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review 47,* 777 – 780.

Freedman, S.J. & Clauser, J.F. (1972). Experimental Test of Local Hidden-Variable Theories. *Physical Review Letters 28*, 938-941.

Kochen, S., Specker, E. (1967). The Problem of Hidden Variables in Quantum Mechanics. *Journal of Mathematics and Mechanics 17*, 59-87.

Kurtsiefer, C., Oberparleiter, M. & Weinfurter, H. (2001). High efficiency entangled photon pair collection in type II parametric fluorescence. *Physical Review A 64*, 023802.

Matsukevich, D.N., Maunz, P., Moehring, D.L., Olmschenk, S. & Monroe, C. (2008). Bell inequality violation with two remote atomic qubits. *Physical Review Letters 100*, 150404-1 - 150404-4.

Rowe, M., Kielpinski, D., Meyer, V., Sackett, C. ,Itano, W., Monroe, C. & Wineland, D. (2001). Experimental violation of a Bell´s inequality with efficient detection. *Nature 409*, 791-794.

Salart, E., Baas, A., Branciard, C., Gisin, N. & Zbinden, H. (2008). Testing the speed of ´spooky action at a distance´. *Nature 454*, 861-864.

Weihs, G., Jennewein, T., Simon, C., Weinfurter, H. & Zeilinger, A. (1998). Violation of Bell´s Inequality under strict Einstein Locality Conditions. *Physical Review Letters 81*, 5039-5043.

Notes:

1. Einstein, A., Podolsky, B. & Rosen, N. (1935). Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review 47*, 777 – 780.
2. Bohm, D. & Ahronov, Y. (1957). Discussion of Experimental Proof for the Paradox of Einstein, Rosen and Podolsky. *Physical Review 108*, 1070 - 1076.
3. Sakurai, J.J. (1994). *Modern Quantum Mechanics.* Reading, Massachusetts: Addison-Wesley Publishing Company, p. 236ff.
4. Quoted form: Einstein, A., Podolsky, B. & Rosen, N. (1935). Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review 47, p.* 780.
5. Quoted form: Schilp, P.A. (1949). *Albert Einstein, Philosopher Scientist*. Evanston, Illinois: Library of Living Philosophers, p. 85.
6. Bell, J.S. (1964). On the Einstein Podolsky Rosen Paradox. *Physics 1*, 195-200.
7. Clauser, J.F.,Horne, M.A., Shimony, A. & Holt, R.A. (1969). Proposed experiment to test local hidden-variable theories. *Physical Review Letters 23*, 880-884.
8. Freedman, S.J. & Clauser, J.F. (1972). Experimental Test of Local Hidden-Variable Theories. *Physical Review Letters 28*, 938-941.
9. Greenberger, D, Hentschel, K., Weinert, F. (2009). *Compendium of Quantum Physics*. Berlin: Springer Verlag, p. 14ff.
10. Aspect, A. , Dalibard, J. & Roger, G. (1982). Experimental Test of Bell's Inequalities Using Time - Varying Analyzers. *Physical Review Letters 49*, 1804-1807.
11. *Compendium of Quantum Physics,* p. 14ff.
12. Weihs, G., Jennewein, T., Simon, C., Weinfurter, H. & Zeilinger, A. (1998). Violation of Bell´s Inequality under strict Einstein Locality Conditions. *Physical Review Letters 81*, 5039-5043.

13. Rowe, M., Kielpinski, D., Meyer, V., Sackett, C. ,Itano, W., Monroe, C. & Wineland, D. (2001). Experimental violation of a Bell´s inequality with efficient detection. *Nature 409*, 791-794.
14. Matsukevich, D.N., Maunz, P., Moehring, D.L., Olmschenk, S. & Monroe, C. (2008). Bell inequality violation with two remote atomic qubits. *Physical Review Letters 100,* 150404-1 - 150404-4.
15. *Compendium of Quantum Physics,* p. 348ff.
16. Kurtsiefer, C., Oberparleiter, M. & Weinfurter, H. (2001). High efficiency entangled photon pair collection in type II parametric fluorescence. *Physical Review A 64*, 023802 .
17. *Compendium of Quantum Physics,* p. 348ff.
18. Salart, E., Baas, A., Branciard, C., Gisin, N. & Zbinden, H. (2008). Testing the speed of ´spooky action at a distance´. *Nature 454*, 861-864.
19. Griffiths, D.J. (1999). *Introduction to Electrodynamics.* Upper Saddle River, New Jersey: Pearson Education, p. 495 ff.
20. Aerts, S., Kwiat, P., Larsson, J. A., & Żukowski, M. (1999). Two-photon Franson-type experiments and local realism. *Physical Review Letters 83*, 2872-2876.
21. Kochen, S., Specker, E. (1967). The Problem of Hidden Variables in Quantum Mechanics. *Journal of Mathematics and Mechanics 17,* 59-87.
22. Von Neumann, J. (1932). *Mathematische Grundlagen der Quantenmechanik. Die Grundlehren der Mathematischen Wissenschaften 38.* Berlin: Springer Verlag.
23. Bohm, D. (1952). A Suggested Interpretation of the Quantum Theory in Terms of `Hidden´ Variables I and II. *Physical Review 85*, 166-193.
24. De Broglie, L. (1926). Sur la possibilité de relier les phénomènes d'interférences et de diffraction à la théorie des quanta de lumière. *Comptes Rendus 183,* 447-448.
25. Quoted from: Bohm, D. (1952). A Suggested Interpretation of the Quantum Theory in Terms of `Hidden´ Variables I and II. *Physical Review 85*, p. 166.
26. Holland, P. R. (1993). *The Quantum Theory of Motion – An account of the de Broglie-Bohm casual interpretation of quantum mechanics*, Cambridge: Cambridge University Press.
27. Bertlmann, R. A., Zeilinger, A. (2002). *Quantum [Un]Speakables – From Bell to Quantum Information*. Berlin: Springer Verlag, p.241 ff.