A Comparison of Equatorial Undercurrent: POP2 Model and Measurement

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An in-depth comparison was conducted comparing descriptions of Pacific equatorial undercurrent of two resolutions of the Parallel Ocean Program model to measurements described in previous papers as well as contemporary data from the TAO/TRITON buoy array. A new model is also proposed to approximate the Galapagos archipelago as a cylinder around which equatorial undercurrent might flow.

It was found that the Parallel Ocean Program model when run at 1° resolution follows measured data measured by Brady and Bryden (1985) as well as from TAO/TRITON much closer than the 0.1° model. Neither model was found to behave correctly when thermal properties were tested, with temperatures decreasing by 3.0°C and 3.7°C between longitudes of 150°W and 110°W, measured previously to be almost isothermal. Also tested are predictions of Ekman effects narrowing the width of equatorial undercurrent at depths above 90m. These effects were found to be present in the 0.1° model but not the 1° model.

Attempts to model the Galapagos archipelago cylindrically were unsuccessful, however the 0.1° model was directly compared to measured results of Karnauskas et al. (2010) and found to be in reasonable agreement of these measurements.

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PREAMBLE

Please note that several of the figures and plots in this contain a lot of detail and might be difficult to see, therefore larger versions of all detailed figures included in this report may be found in the appendices. Furthermore, most of the comparative elements of this investigation rely on the results of Brady & Bryden - "Diagnostic Model of the Three Dimensional Circulation in the Upper Equatorial Pacific Ocean" (1985), in particular, section 4 (Heat Budget). This paper is available online and a link is provided in the references.

I. INTRODUCTION

The Equatorial Undercurrent (EUC), sometimes called the Cromwell Current, is a flow of water found between the depths of 70-170m in the pacific ocean. Flowing from west to east it travels backwards with respect to normal flow of gyres at near-equatorial latitudes ($\pm 1°$N) very quickly, with maximum velocities of over 100cm/s[1][2].
The Parallel Ocean Program (POP2) attempts to describe the kinetics of the ocean, a comparison was done taking the results of POP2 models at $1°$ and $0.1°$ resolutions as well as real data collected from the TAO/TRITON buoy array to test the accuracy of POP2's description of EUC. POP2's description of ocean systems is widespread in oceanographic research and therefore testing its accuracy is important for the validation of research based around the results produced by it.[1]
The description of EUC was first completed by Cromwell et al. (1953)[5] and measured further by Bryden and Brady(1985)[2] who confirmed that cross-isopycnal fluxes were very small, especially at depths below the mixed layer. It was therefore confirmed that the EUC would be sandwiched between isothermal surfaces with maxima found at 20°C. Any test of POP2's description of EUC would have to confirm this as well as compare it to the results of TAO/TRITON.
Also considered is the termination of the EUC and its interaction with the Galapagos islands, lying slightly south of the equator at 91°W $\times$ 0.78°S. In this paper a possible mathematical description of the behaviour of the EUC is given by using an adapted version of creeping flow around a cylindrical surface.
The origin of the EUC is derived from an east-west pressure gradient with several contributing factors. Firstly due to a lack of the Coriolis force at the equator, there is very little meridional current or other diversions to prevent flow along this pressure gradient[1]; zonal velocities are roughly 10 times faster than meridional and naturally vertical velocity is almost 0. A further consideration is that the EUC is seasonal, with a strong season between March and July and a weak season during other months, cycling periodically. This will affect data taken the TAO/TRITON array as we are trying to match annual averages, this must be accounted for in measured data.

II. EXPECTATIONS AND PREDICTIONS

Several studies have been done on the shape and scales of the EUC, as well as mathematical descriptions. We used these to prepare a set of expectations for what results the EUC within the POP2 model should look like and give us a good point of comparison. Listed below are 2 expectations we can predict from previous findings as well as using data freely available from the TAO/TRITON buoy array along the equator.

A. The Profile of the EUC core between 150°W and 110°W along the equator

The profile of the EUC between 150°W and 110°W is well documented and studied as well as described mathematically. This is possible due to the fact we can ignore Coriolis effects very close to the equator as well as ignore Ekman effects at depths below 90m [4]. The profile of velocity along the equator has also been well documented in the past; Bryden and Brady published their profiles of the EUC below (figure 1)[2]:

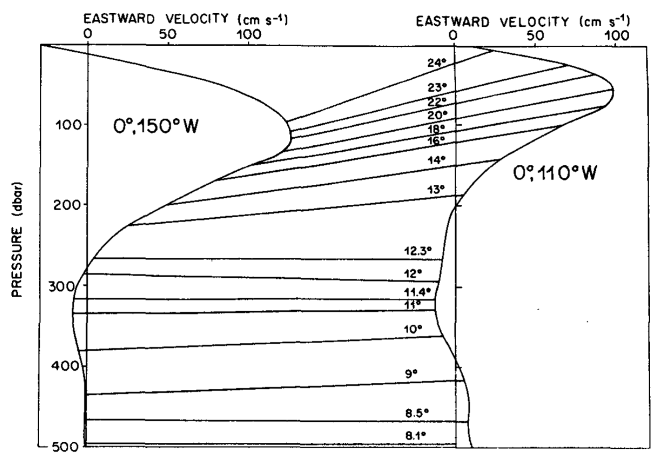


Fig. 1. Figure 12 from Bryden and Brady "Diagnostic Model of the Three-Dimensional Circulation in the Upper Equatorial Pacific Ocean" (1985) plotting zonal velocity profiles as measured at 150°W and 110°W, with lines connecting points of equal temperature
[2]

This profile will give us a target for POP2 to reproduce as well as a point of comparison from modern data from the TAO/TRITON buoy array.

1) A note on Entrainment and Ekman effects: A good mathematical description of Ekman effects on the EUC has not yet been formulated however a heuristic model was devised and tested by Pedlosky (1988)[1] to describe cross-isopycnal mass flux given by:

$$-\int_0^\delta w_* \cdot dy \equiv -M(x) = (v_2 h_2)_{y=\delta} \tag{1}$$

Where M describes the cross-isopycnal mass flux in the latitude band (0,δ) co-responding to the dimensional width $l\delta$. $v_2 h_2$ represents the inward velocity of fluid displaced by ekman effects.
It is then considered that ekman effects will only be significant over eastern regions of the EUC where the current is much closer to the surface. Therefore $M(x)$ is chosen to take the form:

$$M(x) = \begin{cases} 0 & 0 \leq x \leq x_b \\ M_0(\frac{x-x_b}{x_e-x_b}) & x_b \leq x \leq x_e \end{cases} \tag{2}$$

Where x_b represents a longitudinal point along the EUC where Ekman effects begin to have significant effect. This heuristic model of Ekman effects predicts that along the equator itself, the entrainment from either side balance and the velocity remains almost constant along the stream function[1], The heuristic model therefore predicts a thinning and narrowing of the current in order to yield the necessary transport. This effect can also be tested for in the POP2 model.

B. Constant Velocities Along Isotherms.

The equation of zonal momentum (with approximations made for being on the equator), describes the flow of the EUC for layer n (working in dimensionless variables)[1]:

$$[u_n \frac{\partial u_n}{\partial x} + v_n \frac{\partial v_n}{\partial y}] - y v_n = -\frac{\partial p_n}{\partial x} + I \tag{3}$$

Where I represents cross isopycnal fluxes with a common factor of $\frac{w_*}{W}$, Where $W = \frac{U\ H}{L}$. These terms are small, (found by Brady and Bryden to be $O(10^{-5})$ and therefore can be ignored (i.e. $I \approx 0$). Ignoring these terms implies that there are no cross isopycnal fluxes in the EUC. Since isopycs are also isotherms we should expect the maximum zonal velocity of a slice of the EUC to follow and isotherm to any other slice in the model. This prediction was confirmed in Brady and Bryden's paper [2], finding cross isothermal velocities to be $O(10^{-3} cm/s)$ (also visible in figure 1).

C. A Possible Description for EUC Interaction with the Galapagos Islands.

1) Findings from Measurement: The interaction of the EUC with the Galapagos islands has been surveyed therefore the modelling of this interaction by POP2 can also be compared to these findings. As well as this, a mathematical description of flow around the Galapagos islands was attempted with mixed results.
A survey of streamlines flowing around the Galapagos was completed by Karnauskas et al. (2010) and their results[3], which we will use as the point of comparison for the POP2 model are given below (fig 2):

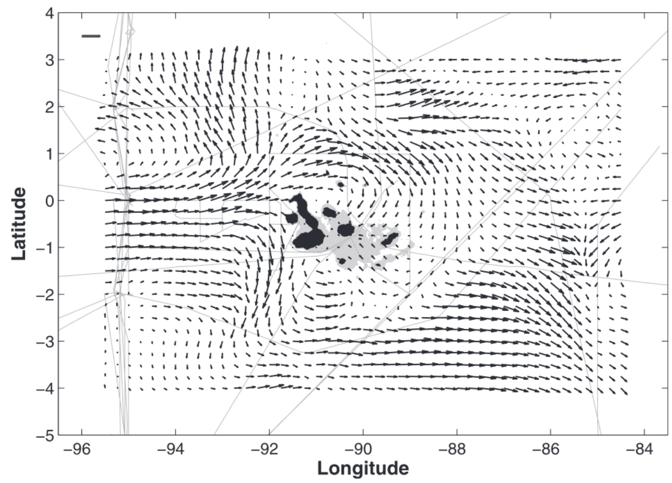


Fig. 2. Figure 5 from "Observing the GalapagosEUC Interaction: Insights and Challenges" - Karnauskas et al. (2010); Plotting depth-averaged horizontal currents from data gathered over 11 cruises (whose routes are plotted in grey). The depth averaging was taken from readings of 40-160m depth. A 25cm/s reference vector is available in the top right. These cruises were conducted during a weak EUC season

Further to this, 5 more cruises were conducted during an EUC strong season (March-July), and the results are available in the paper.

2) Attempt to Model this Behaviour Mathematically: As an attempt to model the flow of currents around, we will attempt to model the Galapagos islands as a cylinder with non-slip boundary conditions. For flow where advective terms can be ignored (i.e. a small Reynold's number), in the absence of external forces, flow around a cylinder centered on the origin is given by:

$$v_r = U cos\ \theta(1 - \frac{3a}{2r} + \frac{a^3}{2r^3}) \tag{4}$$

$$v_\theta = -U sin\ \theta(1 - \frac{3a}{4r} - \frac{a^3}{4r^3}) \tag{5}$$

For a cylinder of radius a and a current with velocity U infinitely far from the cylinder. A short derivation of these equations are given in the appendices. To adjust this to suit EUC, instead of using constant U, we will allow $U \equiv U(x)$ where U will be a curve following zonal velocity along the EUC at all relevant depths (layers of the model). This adjusted model for flow around a cylinder will then be compared to the results both of the POP2 model and the findings of Karnauskas et al. (figure 2)[3].

III. POP2 Model Description and Findings

The POP2 model which was used at 2 resolutions is a subset of the Community Earth System Model (CESM) which attempts to model the earth's ice, sea, land and carbon cycles numerically. The output of this model at 1° and 0.1° resolutions were taken as annual means so any noise or seasonal effects would be filtered out.

A. Shaping of EUC Currents Between 150°W and 110°W

The findings Bryden and Brady show the profile of the EUC between 150°W and 110°W quite clearly (fig 1)[2]. Using the POP2 model at low resolution and high resolution (respective precisions of 1°and 0.1°), we can try to reproduce this graph of currents along the equator by plotting the modelled profiles (fig 3).

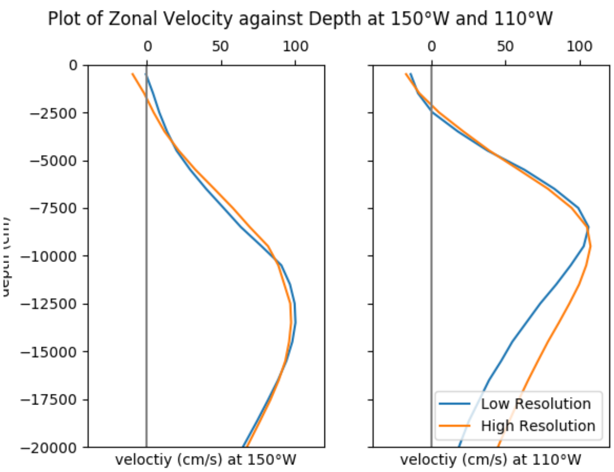


Fig. 3. Model output plotting the modelled velocity cross sections at 150°W and 110°W

Comparing these results firstly of the two resolutions to each other, we can see an immediate difference between the low and high resolutions. Noteably that the predicted surface velocity at 110°W is predicted to be roughly 0cm/s by the low resolution model, when the a current is predicted by the high resolution model. Secondly the lower portion of the EUC falls away much slower in the high resolution model than the low.
We will now compare these two curves with (fig 1):
Firstly, looking at the EUC itself, the maximal zonal velocity has risen from 125m depth to 90m depth, this is in line with and agrees with Brady & Bryden's findings.
Secondly the magnitude of the maximal zonal velocities can be considered and are both inaccurate. The POP2 model finds that the predicted maximal velocity of the EUC will be 100cm/s at 110°W and 110cm/s at 150°W. This is inconsistent with Brady & Bryden who predict a much higher velocity at 110°W and a slightly lower maximum, closer to 100cm/s at 150°W.
Thirdly we can compare surface velocities. Brady and Bryden found a negative zonal velocity at the surface of 110°W and almost 0 zonal velocity at 150°W. In the POP2 model, however, almost the opposite is found. At the surface of 110°W almost 0 zonal velocity is predicted by the lower resolution model and at the surface of 150°W, a negative zonal velocity is predicted.
Finally, the drop-off of the curve at lower depths is predicted well in all cases, dropping to 50cm/s in both resolutions at 110°W, however the model predictions of this drop off is inconsistant, both with eachother and with Brady and Bryden's findings at 150°W. The found result is that the zonal velocity will tend to 0 slightly before 200m depth whereas the model predicts in both cases a positive zonal velocity at this depth for both resolutions, with the low resolution yielding a closer answer to Brady and Bryden's findings.

1) Testing Predicions of Ekman Effects: The predictions of the Ekman effects were that for latitudes north and south of the equator, a thinning should occur of the EUC causing a much soon deceleration at these velocities[1]. To test this hypothesis, the velocity cross section at 110°W and 150°W were also tested at 1°N latitude, to see if this effect was present within the model (fig 4).

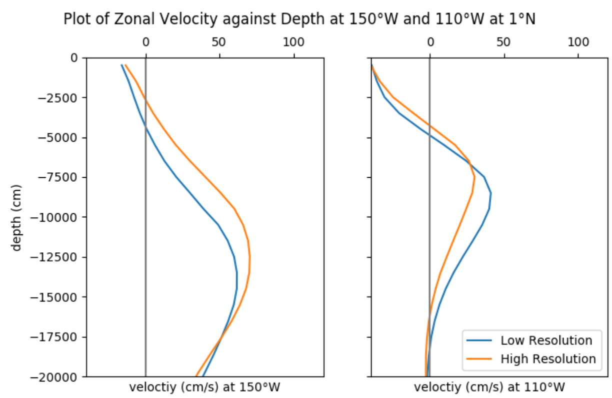


Fig. 4. Model output plotting the modelled velocity cross sections at 150°W and 110°W at 1°N Latitude

There is a significant difference between the two resolutions of the model at this latitude. The low resolution predicts a zonal maximum that remains at 50cm/s whereas the high resolution model decreases in maximal zonal velocity from 65cm/s to 40cm/s. An explaination for this might be that whilst there is only 1 step for the low resolution model between 0°N and 1°N, there are 10 for the high resolution model, allowing enough space for Ekman effects to be modelled successfully.

B. Testing the Prediction of Isothermal Maxima

The aforementioned findings of Byden and Brady found that in the limit that cross isopycnal fluxes can be ignored, which is roughly true in the case of the EUC, the maximum zonal velocity of the EUC follows the isotherm at 20°C.This was tested by finding the modelled maxima and temperatures between 150°W and 110°W longitudes, near the middle of the EUC, where it is normally plotted. The results were found to be inline with Bryden and Brady's findings (See Table I).

High Resolution (0.1°)	Low Resolution (1°)
Mean Temperature: 19.0160°C	Mean Temperature: 19.4391°C
Variance: 1.22465°C	Variance: 0.918125°C
Standard Deviation: 1.10664°C	Standard Deviation: 0.958188°C

TABLE I. MODEL OUTPUT OF TEMPERATURES AT MAXIMAL ZONAL VELOCITY AS WELL AS VARIANCES AND STANDARD DEVIATIONS BETWEEN 150°W AND 110°W LONGITUDES

The temperatures of the models did seem to decrease over time however (Fig.5). This could be explained theoretically by cross isopycnal fluxes however these results are inconsistent with the findings of Bryden and Brady and can therefore be found to be wrong.(refrence figure 12 from brady and bryden). Using regression to approximate these curves linearly, it was found that the low resolution model dropped from 20.9°C to 17.9°C and the high resolution model dropped from 20.9°C to 17.2°C over this longitudinal range.

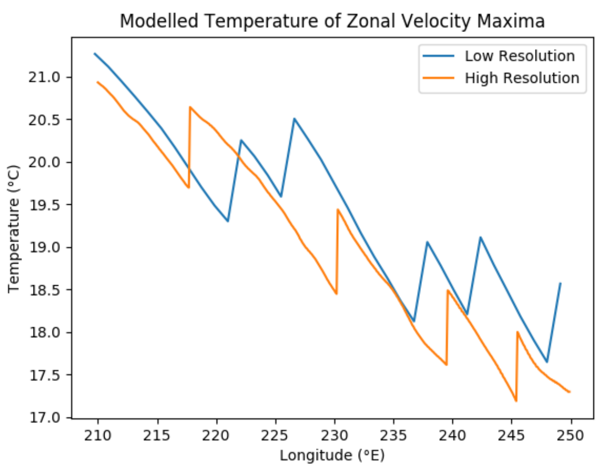


Fig. 5. Model output plotting the temperature of maximal zonal velocity between 150°W and 110°W

The sudden jumps in temperature in this graph represent where the maximal zonal velocity transfers from 1 layer to another within the POP2 model, if the maximum were to follow isotherms perfectly, we would expect these jumps all to lead to the same temperature however this is not the case. This may be explained to a certain extent by cross-isopycnal fluxes mentioned in section A however that is insufficient to explain this well. In Brady and Bryden's 1985 paper, the cross-isothermal velocities were found to be (table II):

Temperature (°C)	Cross Isothemal Velocity (cm/s $\times 10^{-3}$)
21	0.44
20	0.29
19	0.11
18	0.08
17	0.02

TABLE II. CROSS ISOTHERMAL VELOCITIES AT DIFFERENT TEMPERATURES AS MEASURED BY BRADY AND BRYDEN, THESE RESULTS ARE TAKEN FROM TABLE 3 "DIAGNOSTIC MODEL OF THE THREE-DIMENSIONAL CIRCULATION IN THE UPPER EQUATORIAL PACIFIC OCEAN" (1985)

These results imply that cross isopycnal velocities are too small to account for these changes in temperature and that the POP2 model is inconsistent with realistic results.

C. *POP2 Interaction with the Galapagos*

To test POP2's interaction with the Galapagos, only the high resolution model was used. The reason for this is that the radius of the Galapagos islands in their entirety is approximately 0.9°and within a 1°model, very little information is available for comparison with Karnauskas et al.'s findings.

For the most direct comparison, an average was taken of horizonal velocities predicted by the POP2 between layers 4 and 16 (depths 40 - 165m), the result of this averaging is plotted below (figure 6):

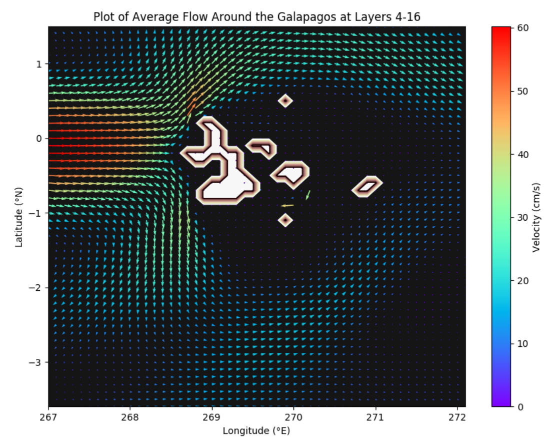


Fig. 6. Plot of average currents around the Galapagos between layers 4 and 16 (depths 40 - 165m) with absolute velocities denoted both by colour and size of the arrows

We can now begin to compare POP2 with Karnauskas et al.'s results[3]. There are some immediate differences to notice between the 2 plots. Firstly the POP2 model predicts much faster absolute velocities than were found by Karnauskas et al. however this is because POP2 was predicting annual means whereas Karnauskas et al.'s data was taken during a weak season of the EUC therefore this is not a fair comparison to make; further depth-averaged measurements have also been made with velocities recorded of 50-60cm/s during the strong EUC season with fewer measurements. A better point of comparison is the shape of the current. Overall there is a very good agreement between POP2 and Karnauskas et al.'s measurements for the general shape of the bow of the current accurately predicting motion as far as 269°E. After this point however, there is a significant difference between the POP2 model and measurement. It is predicted by the model that at 271°E both of the diverged currents flowing around the islands have recoupled and are continuing eastwards. Measurement however would suggest that flow at this point is southward with

the reality of this point being that the flows are very turbulent with several eddy currents flowing, particularly around the Marchena and Floreana islands. The predicted currents in the area between the islands in the POP2 model does not show any of this turbulence at all, on the contrary it predicts almost no current between the islands at all. This however is also interesting to note, as individual layers of the POP2 model do have currents flowing between the islands however since different layers predict different currents these almost cancel out entirely. An example of this is given below, plotting the individual data for layers 5, 10 and 15 (figure 7):
Despite the fact that cross currents between Santa Cruz and San Cristobál are found in individual layers of the POP2 model, they predict absolute values of these currents to be 15cm/s. Although this is similar to the measured values of average cross current, the measurements were made during a weak season of the EUC so even these predicted values are likely to be too small.

IV. Comparison to Recent Results from TAO/TRITON

Further to simply comparing the two resolutions of models to results of other surveys, we can use data output from the TAO/TRITON buoy array, a set of buoys along the pacific equatorial region measuring velocity profile using acoustic doppler profilers to measure the current at multiple depths. Data from this array was used and plotted in superposition with the POP2 model outputs below:

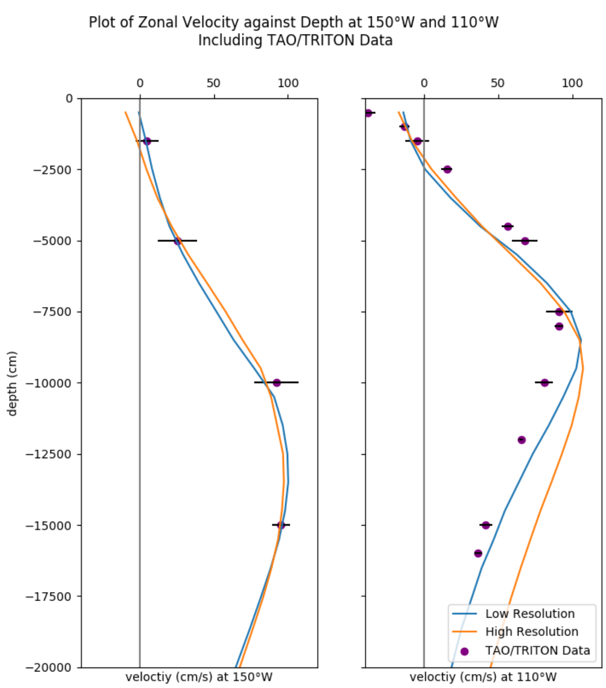


Fig. 8. Model output plotting the temperature of maximal zonal velocity between 150°W and 110°W including data from the TAO/TRITON array

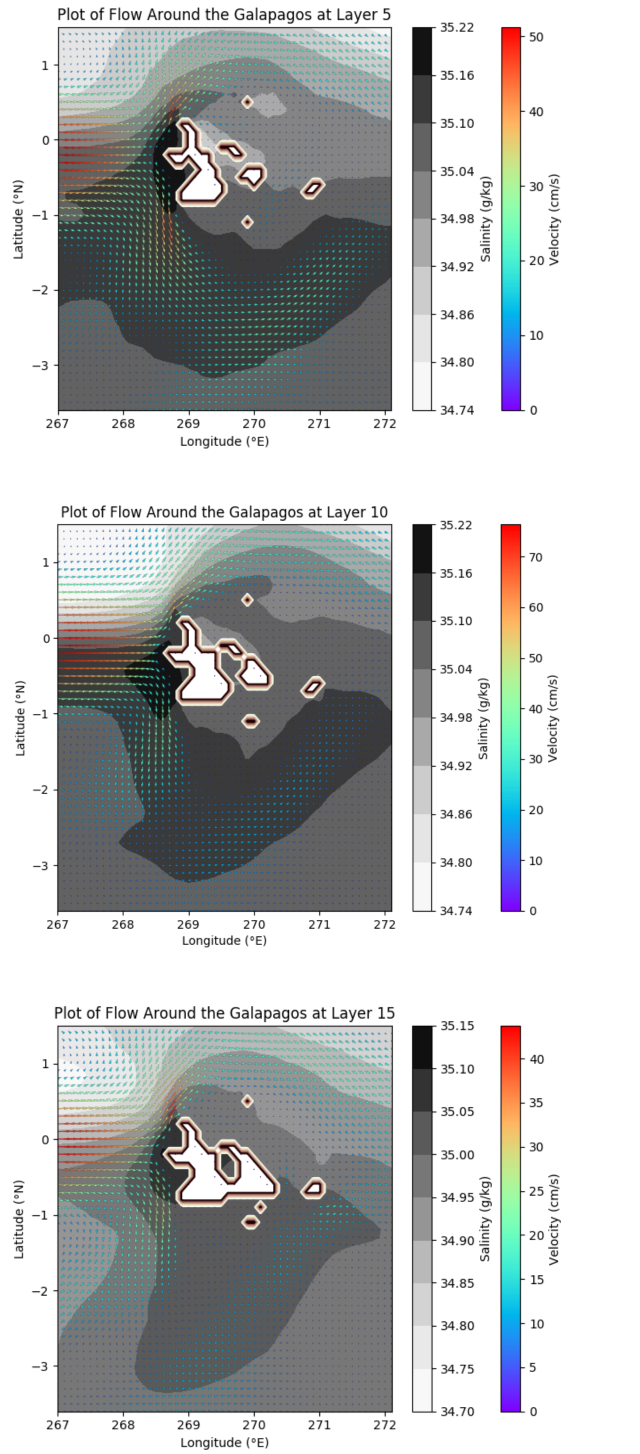


Fig. 7. Plot of currents around the Galapagos as predicted by the POP2 model showing especially the different directions of currents flowing between the islands.

Data series from TAO/TRITON used was formatted in series where each point was separated by 30 julian days however also represented an average of 3 months of data. This meant that each reading, in a manner of speaking was triple counted. Whilst this wouldn't affect the means themselves in calculating induvidual points, it did mean calculated standard deviations were too small by a factor $\sqrt{3}$. This was accounted for and appropriately adjusted for before plotting.

We can now compare these results both to the model and to Brady and Bryden's findings. The most obvious point to notice is that there are far fewer data points for 150°W than for 110°W. This is due to the fact that this data was collected by an older mooring, using vector averaging methods and mechanical meters. Since the data at this point was collected only between 1980-1995, the uncertainty on measurements taken at this longitude are also less certain. The data that is there at this longitude, however, is very close to POP2's predictions, with the low resolution model being closer to reality. Since there is little data around the peak current however, it is difficult to say whether this data agrees with the findings of Brady and Bryden that the maximal zonal velocity will be well over 100cm/s. It also disagrees with Brady and Bryden's findings for surface velocity however that there will be a significant negative zonal velocity at the surface along this latitude, instead following the low resolution POP2 prediction that zonal velocity will be small at this point.

Now we will consider 110°W longitude. Here TAO/TRITON's results deviate from both resolutions of the POP2 model. Firstly there is an anomalous data point at 5m depth. This point deviates from the other points from TAO/TRITON, both resolutions of POP2 and Brady and Bryden's findings. Looking at the individual data for this point, it is only time averaged from 5 individual points taken over 7 months in 2005, during a strong season of the EUC. Therefore this point can be considered anomalous and disregarded as it may easily have been skewed by noise (discussed in the next paragraph). All other points at this longitude have between 33 and 260 months of data. Secondly before the maximum point of zonal velocity, the velocity is higher than POP2's prediction and after the maximum it is lower. This may imply a higher depth of the zonal maximum at this point however since an exact mathematical description of this curve is unknown, it cannot be fit to the data, instead only compared to model output. Furthermore, as the data tends towards 200m depth, the zonal velocity approaches closer to 0 much more than either resolution of the POP2 model. A measurement that can be made however is that at 55m depth at this longitude, the predicted temperature is 21.0°C. If the maximal zonal velocity were to follow an isotherm as suggested by Brady & Bryden, the agreement between POP2 and TAO/TRITON measurements would be improved.

Data from TAO/TRITON is very noisy with the range of data contributing to each point on fig 8 being over 150cm/s. An example of this is given below of the data at 110°W at 80m depth and 45m depth (figure 9):

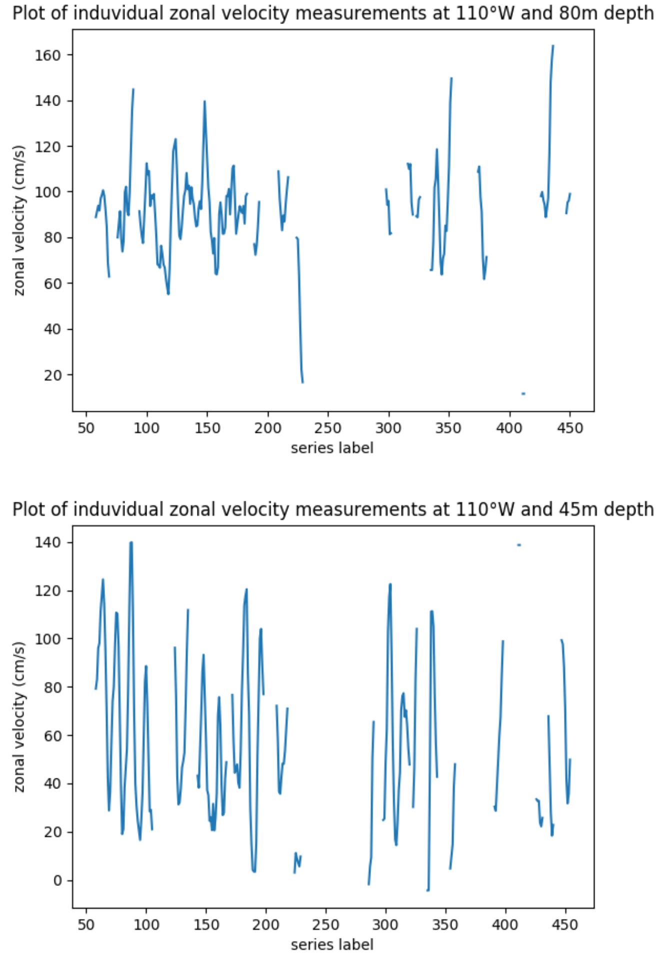


Fig. 9. Plot of induvidual measurements taken by a TAO/TRITON buoy at 110°W at 80m depth and 45m depth to demonstrate the noise associated with measurements of the EUC. Each point along the time series represents 30 julian days measured from Jan 16, 1979.

This noisiness of the data may explain the difference in the findings between TAO/TRITON's readings and the measurements taken by Bryden and Brady. The measurements used in their paper were based on 6 years of data collection by the Hawaii to Tahiti Shuttle Experiment and the Equatorial Pacific Ocean Climate Study (EPOCS). Any time-averaged measurements made in this survey may have been skewed by noise. Since this comparison has used data spanning from 1979 to 2017, we can be more confident in results gathered. The reason for this noise is that the flow of EUC data is seasonal with a strong season between March and July and otherwise being weaker. Further noise may be added by interaction with El Nino a process known to cause upwelling in the EUC where it meets the Galapagos.

V. Modelling Flow around the Galapagos as Stokes Flow around a Cylinder

We will now attempt to model the flow of the EUC around the Galapagos islands as stokes flow around a cylinder (described by equations 4 & 5). This describes flow of viscid liquid moving around a cylinder where advective terms can be ignored (i.e. for a small Reynold's number) and where flow far away from the cylinder is constant. If we try to approximate the flow of the EUC in this way, we must adjust $U \equiv U(x)$. To find the functional form of U at any layer, we will use regression to a 4th order polynomial applied to the zonal velocities of each layer of the low resolution POP2 model.

An example of this plot can be seen in figure 10 before regression is applied. As was found in section 4 of this paper, the low resolution model fits much closer to real data collected by TAO/TRITON. It is therefore this model that was chosen to apply regression to. The regression was calculated for all layers between 5 and 19 in the model, fitting with R^2 values ranging between 0.9092 and 0.9981, a full list of these equations and their fits can be found in appendix B. The quality (R^2) value of the regression plots increases significantly at the lower layers. This is because the velocity curves are more turbulent for the higher layers as they approach the mixing layer and more complicated sheering processes begin as the surface zonal velocity is negative.

Now we have found a functional form for U, we can must find an approximate form of a cylindrical Galapagos. To do this we can look at the flow of the EUC in the POP2 model (figure 6). With this in mind, the approximate form chosen was that the Galapagos would be a cylinder with radius 0.9°centred at 269.5°E and 0°N. This is obviously not the most realistic location however centering the cylinder along the stream of the EUC may work as a first approximation. The result of using this centring and approximation of can be seen below (figure 11)

Comparing this to the POP2 model, it is clearly not a perfect model. We can however make some comparisons between the models. Firstly, for higher layers of the POP2 model, stokes flow models the shape of currents around the islands reasonably well. Comparing the 5th layers, the current correctly diverges and rejoins around the islands however the flow around the modelled cylindrical islands rejoins much faster than predicted by POP2.

There are further problems with this model of the Galapagos. One such problem is that the modelled streamline flowing along the equator (θ = 0) follows the predicted streamline:

$$\mathbf{v}(x) = U(x)(1 - \frac{3r_g}{2x} + \frac{r_g^3}{2x^3})$$

Where x represents the longitude, normalised to where the Galapagos can be found on the 0°meridian. r_g represents the modelled radius of the cylindrical Galapagos. This predicted stream slows down too soon in the cylindrical model compared to POP2 as well as measurement. The cylindrical model predicts that flow will have slowed to as low as 15 cm/s at a distance of 2°from the front of the first island. This is incorrect as the current 0.2°before the island as predicted by POP2 and measured by Karnauskas et al. is roughly 30cm/s.

Looking at this problem more generally there is a problem that all velocities predicted in the cylindrical model are too small. The model is plotted in a 4°×4°square around the modelled cylinder and the maximum velocity predicted is approximately 30cm/s. Measurement by Karnauskas et al. would suggest this could be acceptable for a weak season however since this is modelled from data taken as an annual average, this velocity should be expected to be larger for this model.

Whilst the above comparison is true for both the 5th and 10th layers, there are even more severe problems comparing the 15th layer to the cylindrical model. At this depth the entire shape of the predicted currents is incorrect. POP2 predicts, that currents at this point will turn around completely and continue westwards, a behaviour that is not predicted at all by the cylindrical model. Flow after the archapaelago is then provided by a current approaching from the south.

The reason that the model of the Galapagos as Stokes flow around a cylinder is so bad lies in assumptions made along the course of the model. Firstly we are assuming that U will be a laminar flow towards the cylinder, this is incorrect as we have seen already that the EUC is approximately 2°in width and gets thinner as it continues as a direct result of Ekman effects. By assuming a laminar flow approaching the cylinder, we preclude the possibility of flow reversing direction as is the case demonstrated here by looking at the 15th layer. Secondly we have assumed without proper justification that Reynold's number will be small, essentially forbidding any complicated motion such as eddies or vorticies; this will become especially true as the considered distance ahead of and behind the cylinder increases. Thirdly we have shown in the appendices the functional form of v assuming constant U, however we have then afterwards adjusted $U \to U(x)$ without rederiving the equations of motion for flow around a cylinder. Since these are derived from a partial differential equation in x, simply assuming that $U \to U(x)$ is naive.

A further assumption made in the derivation of this flow is found in the boundary conditions. We have employed no-slip boundary conditions in this derivation, which by inspecting realistic flow around the Galapagos both in the POP2 model and Karnauskas et al.'s results is clearly wrong. It can be seen that currents flow next to the Galapagos with velocity greater than 0, therefore a more accurate model may be found in future by employing different boundary conditions such as semi-slip allowing some movement along the surface of a modelled cylinder.

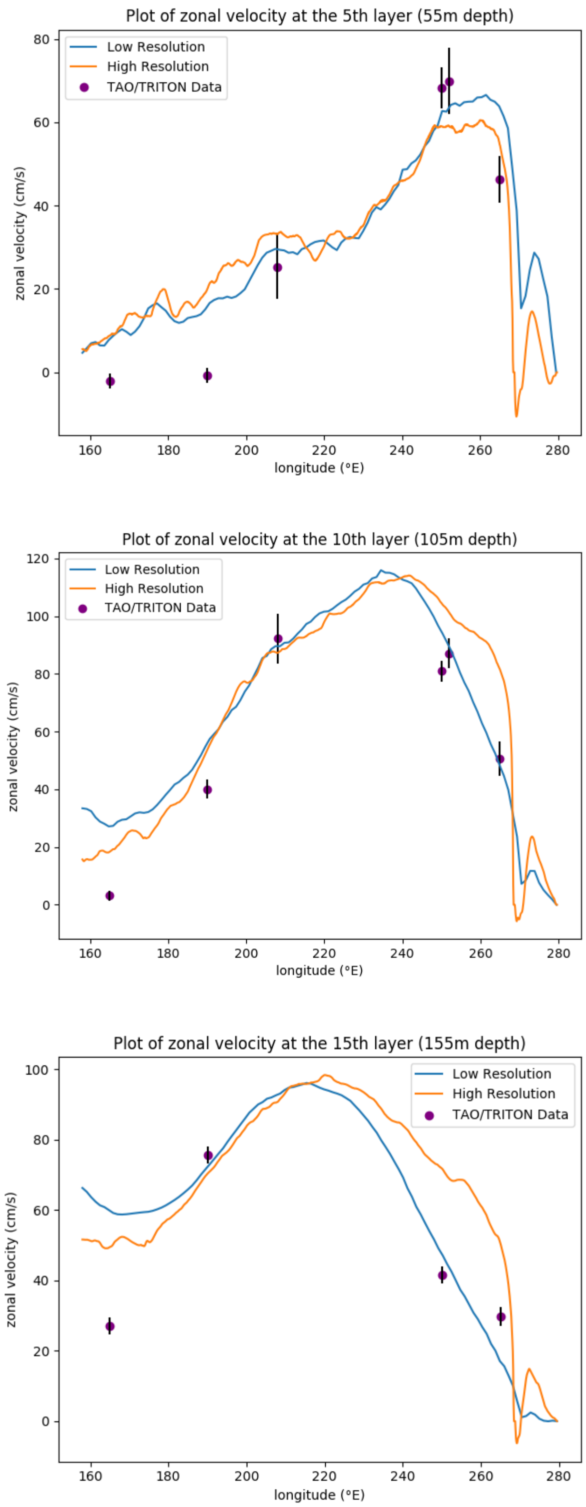


Fig. 10. Plot of zonal velocity of the EUC as a function of latitude as predicted both by the low and high resolution models as well as TAO/TRITON's data at the 5th, 10th and 15th layers

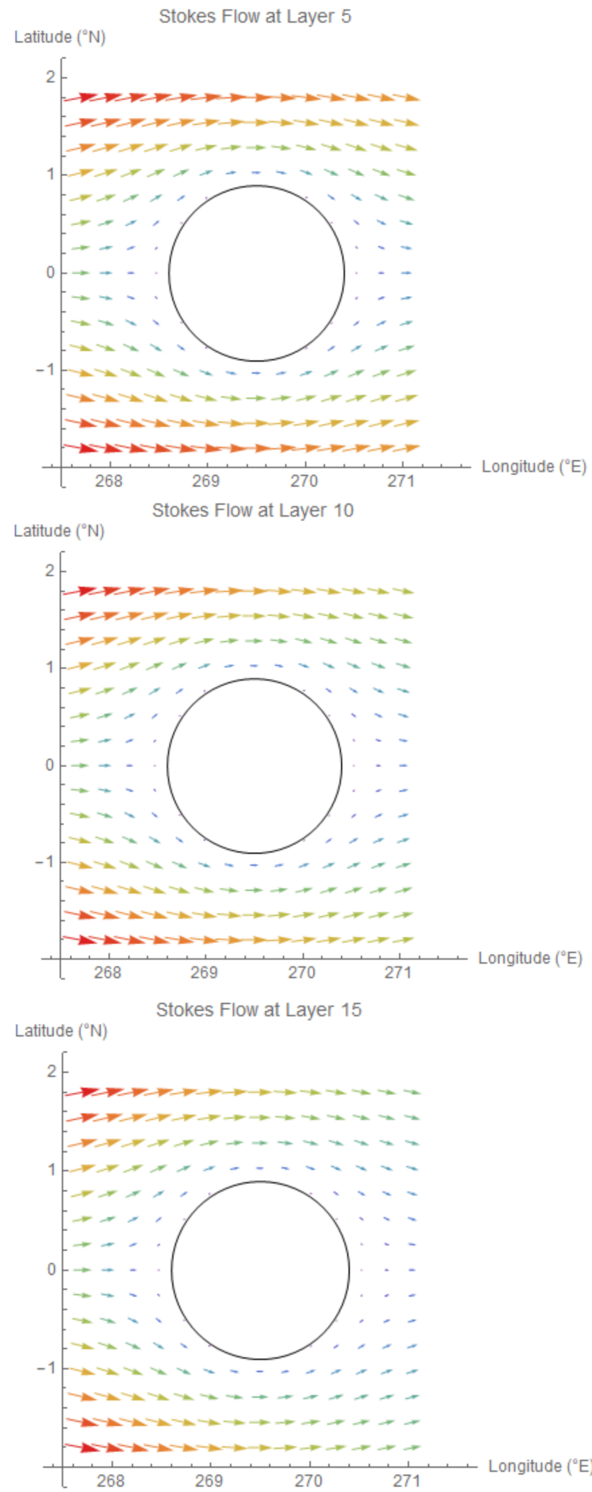


Fig. 11. Plot of approximated stokes flow around the Galapagos for the 5th, 10th and 15th layers as approximated cylindrically

VI. Conclusion

The main conclusion to draw from this comparison is that whilst the POP2 model predicts the scale and shape of the EUC reasonably well there is still much room for improvement especially with its high resolution model. A conclusion of each individual point of comparison offered in this paper is given below:

A. General Shape of the EUC

The main discrepancy between the POP2 prediction and the measured values of the EUC are that the zonal velocity maxima for both resolutions for the model are too deep by approximately 15m as seen in figures 6 and 1. This effect is very closely linked to the face that the modelled maxima does not follow the predicted 20°C isotherm, instead falling to a cooler isotherm over this longitudinal range, shown in this case by the maximal zonal velocity dropping. It is concluded that a significant part of cause of this problem is that the maximal zonal velocity does not follow an isotherm as predicted by Bryden & Brady.
The second problem is that velocities do not decrease fast enough for deeper layers of the model when compared to experiment, especially in the case of 0.1°resolution.

B. Isothermal Maxima of Zonal Velocity

It was seen in this comparison that maxima of zonal velocity did not follow an isotherm as was predicted. A reason for this maybe that modelled vertical mixing within the POP2 model considers the Richardson number and K-profile parameterisation, these may not properly account for the parameterisation offered by Brady & Bryden's diagnostic model, this should be considered as an addition for editions of the POP2 model, perhaps as a conditional expression.
The effect of temperature in diffusivity within the POP2 model is given by

$$\kappa_d = VISCM \times 0.909\ exp(4.6\ exp[-0.54(R_\rho^{-1} - 1)])$$

Where $VISCM$ represents the molecular viscoscity and R_ρ represents the density ratio between points on the tracer grid modelled within the POP2 model[6]. This diffusivity makes up a single part of four within the total modelled diffusivity, however it may need to be adjusted to fit observed data as has been done for the salt fingering regime to provide a perfect model of EUC.

C. Modelling Flow around the Galapagos as Creeping Flow

Modelling the Galapagos islands and the EUC interaction with them as creeping flow around a cylinder was a complete failure within the model offered in this paper. If it were to be attempted again, it would have to be a form completely rederived from the Navier-Stokes equation to allow $U \equiv U(x)$ from the very beginning of its derivation. Beyond that several of the boundary conditions must be reevaluated, Non-slip boundary conditions appear to be unphysical for application here because most of the modelled circle, in reality, is additional seawater against which it doesn't make sense to say $v_\theta(a, \theta) = 0$. The assumptions that do hold are that seawater is roughly uncompressable; we may ignore both the corriolis forces and bottom friction. If it were to be done again, a more in-depth look into both the Reynold's number and by extension advective terms in within the derivation would be recommended for better results.

D. Final General Conclusions

In this paper, comparisons were drawn between 2 resolutions of the POP2 model and real life measurements, both previously published and acquired independently from TAO/TRITON. For almost every test given to the POP2 model, it was found that low resolution model working at 1°precision gave closer results to reality than the high resolution model working at 0.1°precision. Exceptions to this rule were found when testing Ekman effects on the EUC, when testing the latitude 1°N, the Ekman effects were found to be present in the high resolution model but not the low resolution model. A reason for this could be that for the Low resolution model, 1°N is only 1 step away from the core and the Ekman effect doesn't have enough space to manifest at this scale. Overall, the POP2 model needs to evolve as it has done in the past by directly setting its parameters to match experimental measurement of the oceans as more modern and reliable data is collected.

VII. Post Scriptum

The following measurements were made but considered unhelpful for comparison between TAO/TRITON measurements and the POP2 model: For figure 8, the average number of standard deviations was calculated for the two available longitudes, this was considered a useless statistic because instead of trying to fit data to a theoretical curve, we are fitting the output of a model to data and so this statistic is not relevant or useful. The measurement at 5m depth 110°W latitude was ignored for this calculation as was considered anomalous from insufficient data (only 5 months worth of readings were available this point).

Longitude (°W)	LR Mean # STD (cm/s)	HR Mean # STD (cm/s)
140	0.18	0.38
110	2.60	4.29

The exact functional form of first order regression plots of figure 5 was calculated and used but not explicitly stated as it was considered irrelevant for all points besides extremal.

Resolution of Model	Functional form of regression line
1°	T = 36.393 - 0.0739L
0.1°	T = 40.358 - 0.0928L

For longitude L and temperature T.

APPENDIX A
A SHORT DERIVATION OF CREEPING FLOW AROUND A CYLINDER

The Navier-Stokes equation describing the movement of a packet of incompressable fluid with volume dV can be written:

$$\rho\frac{\partial \mathbf{v}}{\partial t} + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \mu\nabla^2\mathbf{v} + \mathbf{F} \tag{6}$$

We will use this to describe the flow of seawater around a cylinder, radius a which we will use as a model of the Galapagos islands.
Where F represents external forces.
If we assume that we can ignore the advective term, which is acceptable given a sufficiently small Reynold's number, the equation will simplify by removing non-linearity.
Reynold's number is given by $Re = \frac{Lu\rho}{\mu}$, significantly smaller than unity for the case we are considering with sea water on the scale of the Pacific Ocean.
We will also assume $F = 0$. This ignores certain forces usually included in oceanography, namely; Coriolis Force, minimal at the equator; and Bottom Friction as we are considering layers well above the bottom;
Therefore in the absence of body forces, we can rewrite the Navier-Stokes equation for a steady-state flow:

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \mu\nabla^2\mathbf{v} = 0 \tag{7}$$

We can search for a solution to this equation using cylindrical polar co-ordinates, assuming $\mathbf{v}$ will take the form $\mathbf{v} = (v_r(r,\theta), v_\theta(r,\theta), 0)$. If we now attempt to solve this equation by separation of variables, allowing:

$$v_r(r,\theta) = R_r(r)\cdot\ \Theta_r(\theta)$$

$$v_\theta(r,\theta) = R_\theta(r)\cdot\ \Theta_\theta(\theta)$$

Applying the change of variables to the Navier-Stokes equations, allowing for the variable-separated form of v we have assumed, we can find:

$$0 = -\frac{\partial p}{\partial r} + \mu(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial v_r}{\partial r}) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}(sin\theta\frac{\partial v_r}{\partial\theta}) - \frac{2v_r}{r^2} - \frac{2v_\theta}{r^2}cot\theta - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}) \tag{8}$$

$$0 = -\frac{1}{r}\frac{\partial p}{\partial\theta} + \mu(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial v_\theta}{\partial r}) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}(sin\theta\frac{\partial v_\theta}{\partial\theta}) - \frac{v_\theta}{r^2 sin^2\theta} - \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}) \tag{9}$$

To solve these Partial differential equations, we must use boundary conditions. We assume Non-slip conditions at the surface of the cylinder:

$$v_\theta(a,\theta) = 0$$

We assume as well that at a sufficiently large distance away from the cylinder, water will flow at a constant rate U:

$$(v_r(\infty,\theta), v_\theta(\infty,\theta)) = (Ucos\theta, Usin\theta))$$

Finally we will also employ the continuity equation for an incompressible fluid in polar co-ordinates:

$$\frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{rsin\theta}\frac{\partial(sin\theta\ v_\theta)}{\partial\theta} = 0$$

Solving (8) and (9), we can find the final solutions for the motion of fluid around a cylinder as well as its associated pressure field:

$$v_r = Ucos\ \theta(1 - \frac{3a}{2r} + \frac{a^3}{2r^3}) \tag{10}$$

$$v_\theta = -Usin\ \theta(1 - \frac{3a}{4r} - \frac{a^3}{4r^3}) \tag{11}$$

$$p = p_0 - \frac{3\mu\ Ua}{2r^2}cos\ \theta \tag{12}$$

Appendix B
Tables of approximations for POP2 at different layer depths

The below tables give 4th order polynomial curve approximations to the POP2 model output for zonal velocity at different layer depths, as well as each curve's R^2 value. These curves were plotted using the low resolution (1°) model as its output was found to be more accurate. The curves are valid using co-ordinates with longitude centred on the Galapagos (i.e. $x = x - 274$ in degrees east) and are approximated between 160°E and 280°E.

Layer	Depth (cm)	Curve Equation	R^2
5	5500	-1E-05x^4 + 0.0019x^3 - 0.1055x^2 + 2.4588x - 3.8592	0.9092
6	6500	-1E-05x^4 + 0.0022x^3 - 0.1232x^2 + 2.9485x - 2.9633	0.9328
7	7500	-1E-05x^4 + 0.0018x^3 - 0.0904x^2 + 2.3446x + 3.5621	0.9504
8	8500	-8E-06x^4 + 0.001x^3 - 0.0367x^2 + 1.4201x + 10.379	0.9639
9	9500	-9E-07x^4 - 0.0004x^3 + 0.0584x^2 - 0.4785x + 22.431	0.9765
10	10500	5E-06x^4 - 0.0017x^3 + 0.1407x^2 - 2.2245x + 37.994	0.9854
11	11500	1E-05x^4 - 0.0027x^3 + 0.2059x^2 - 3.8621x + 55.765	0.9868
12	12500	1E-05x^4 - 0.0031x^3 + 0.2305x^2 - 4.4209x + 66.537	0.9916
13	13500	1E-05x^4 - 0.0031x^3 + 0.2216x^2 - 4.1912x + 71.57	0.9946
14	14500	1E-05x^4 - 0.0029x^3 + 0.1997x^2 - 3.6352x + 72.784	0.9958
15	15500	1E-05x^4 - 0.0026x^3 + 0.1726x^2 - 3.0475x + 72.679	0.9976
16	16510	1E-05x^4 - 0.0023x^3 + 0.1488x^2 - 2.5455x + 71.301	0.9975
17	17548	9E-06x^4 - 0.002x^3 + 0.123x^2 - 2.0182x + 68.458	0.9981
18	18629	8E-06x^4 - 0.0017x^3 + 0.1013x^2 - 1.5975x + 65.076	0.9979
19	19766	7E-06x^4 - 0.0014x^3 + 0.0777x^2 - 1.1192x + 59.896	0.9979

TABLE III. Table of curves approximating zonal velocity in the Pacific for different layers of the POP2 model at 1° resolution and lattitude of 0°N

Appendix C
Mathematica Code Used to Generate Cylindrical Model of the Galapagos

```
a = 0.9;
U = -6 * 10^-6 x^4 - 0.0014 x^3 - 0.0972 x^2 - 1.6433 x + 52.549;
x0 = 265;
field = {
   U Cos[\[Theta]] (1 - (3 a)/(2 r) + a^3/(2 r^3 )), -
     U Sin[\[Theta]] (1 - (3 a)/(4 r) - a^3/(4 r^3))};
cfield = TransformedField["Polar" -> "Cartesian",
     field, {r, \[Theta]} -> {x, y}] - {0, 0};
cfield = cfield /. x -> (x - x0);
cond  = ConditionalExpression[cfield, (x - x0)^2 + y^2 > a^2];
A = VectorPlot[cond, {x, x0 - 2 a, x0 + 2 a}, {y, -2 a, 2 a},
    VectorColorFunction -> "Rainbow", Frame -> False , Axes -> True,
    AxesOrigin -> {x0 - 2, -2},
    AxesLabel -> {"Longitude (\[Degree]E)", "Latitude (\[Degree]N)"},
    PlotLegends -> Automatic];
B = Graphics[{White, EdgeForm[Thin], Disk[{x0, 0}, a]}];
Show[A, B]
```

Appendix D
Python Code Used to Generate Flow around the Galapagos From POP2 Data

The following code was used to generate a quiver plot of flow around the Galapagos. In order to run this code the following modules must be imported:

```
import numpy as np
from netCDF4 import Dataset
import matplotlib.pyplot as plt
```

```
def galapagos_arrows(layer, var = 'SALT', resolution = 7):
    nc = Dataset('ctrl.g.e11.G.T62_t12.002.pop.h.UVEL.0026.annualmean.nc','r')
    nc2 = Dataset('ctrl.g.e11.G.T62_t12.002.pop.h.VVEL.0026.annualmean.nc','r')
    if (var == 'SALT'):
        nc3 = Dataset('ctrl.g.e11.G.T62_t12.002.pop.h.SALT.0026.annualmean.nc','r')
    elif (var == 'TEMP'):
        nc3 = Dataset('ctrl.g.e11.G.T62_t12.002.pop.h.TEMP.0026.annualmean.nc','r')
    else:
        print('Input error')
        return

    U = nc.variables['UVEL'][0,layer,1144:1195,166:218]
    V = nc2.variables['VVEL'][0,layer,1144:1195,166:218]
    background = nc3.variables[var][0,layer,1144:1196,166:218]
    mask = nc3.variables[var][0,layer,1144:1196,166:218]
    nc2.close()
    print("units:",nc3.variables[var].units)
    nc3.close()
    print("layer depth:",np.array(nc.variables['z_t'])[layer],"cm")
    nc.close()
    U = np.array(U)
    U[U>1e30] = np.nan
    V = np.array(V)
    V[V>1e30] = np.nan
    background = np.array(background)
    mask = np.array(mask)
    mask[mask < 1e30] = 0
    mask[mask > 1e30] = 1
    background[background > 1e30] = np.nan
    X,Y= np.mgrid[0:52,0:52][::-1]
    X += 2670
    X = X/10
    Y -= 36
    Y = Y/10
    H = np.hypot(U,V)
    plt.figure()
    plt.contourf(X,Y,background,resolution,cmap = 'Greys')
    plt.contour(X,Y,mask,cmap = 'pink_r')
    plt.colorbar(plt.quiver(X,Y,U,V,H,units = 'width', cmap = 'rainbow',
                    edgecolor = 'k',linewidth = 0.1)).set_label('Velocity (cm/s)')
    plt.colorbar(plt.contourf(X,Y,background,resolution,
                    cmap = 'Greys')).set_label('Salinity (g/kg)')
    plt.quiver(X,Y,U,V,H,units = 'width', cmap = 'rainbow',edgecolor = 'k',
                    linewidth = 0.1)
    plt.xlabel('Longitude ( E )')
    plt.ylabel('Latitude ( N )')
    plt.savefig('arrows.png',dpi = 1000)
    return plt.show()
```

APPENDIX E
PYTHON CODE USED TO GENERATE PLOT OF ZONAL VELOCITY PROFILES AT 150°W AND 110°W FROM POP2 DATA AND TAO/TRITON DATA IN SUPERPOSITION

The following code was used to generate a line plot of zonal velocity against depth at 2 different longitudinal locations, taking the mean of data from TAO/TRITON's output. In order to run this code the following modules must be imported:

```
import numpy as np
from netCDF4 import Dataset
import matplotlib.pyplot as plt
```

```
def zonal_velocity_plot():
    f,(ax1,ax2) = plt.subplots(1,2, sharey=True)
    a,b,c = 221,3199,5
    nc = Dataset('ctrl.g.e11.G.T62_gx1v6.002.pop.h.0400.annualmean.nc','r')
    u = nc.variables['UVEL'][0,0:21,186,a]
    z = nc.variables['z_t'][0:21]
    nc.close()
    u = np.array(u)
    z = -np.array(z)
    nc = Dataset('ctrl.g.e11.G.T62_t12.002.pop.h.UVEL.0026.annualmean.nc','r')
    u2 = nc.variables['UVEL'][0,0:21,1181,b]
    z2 = nc.variables['z_t'][0:21]
    nc.close()
    u2 = np.array(u2)
    z2 = -np.array(z2)
    nc = Dataset('cur_xyzt_qrt.cdf',"r")
    u3 = []
    z3 = []
    e = []
    for i in range(17):
        temp = nc.variables['U_320'][c,0,i,:]
        temp = temp[temp < 1e30]
        if(len(temp)>0):
            u3 += [np.mean(temp)]
            z3 += [nc.variables['depth'][i]]
            e += [(np.std(temp))/(np.sqrt(len(temp)))]
    nc.close()
    z3 = [-100*x for x in z3]
    ax1.plot(u,z)
    ax1.plot(u2,z2)
    ax1.errorbar(u3,z3,0,e,color = 'black',linestyle = 'none')
    ax1.scatter(u3,z3,35,color = 'purple')
    ax1.axis([-40,120,-20000,0])
    ax1.xaxis.tick_top()

    a,b,c = 257,3599,10
    nc = Dataset('ctrl.g.e11.G.T62_gx1v6.002.pop.h.0400.annualmean.nc','r')
    u = nc.variables['UVEL'][0,0:21,186,a]
    z = nc.variables['z_t'][0:21]
    nc.close()
    u = np.array(u)
    z = -np.array(z)
    nc = Dataset('ctrl.g.e11.G.T62_t12.002.pop.h.UVEL.0026.annualmean.nc','r')
    u2 = nc.variables['UVEL'][0,0:21,1181,b]
    z2 = nc.variables['z_t'][0:21]
    nc.close()
    u2 = np.array(u2)
    z2 = -np.array(z2)
```

```
nc = Dataset('cur_xyzt_qrt.cdf',"r")
u3 = []
z3 = []
e = []
for i in range(17):
    temp = nc.variables['U_320'][c,0,i,:]
    temp = temp[temp < 1e30]
    if(len(temp)>0):
        u3 += [np.mean(temp)]
        z3 += [nc.variables['depth'][i]]
        e += [(np.std(temp))/(np.sqrt(len(temp)))]
nc.close()
z3 = [-100*x for x in z3]
ax2.plot(u,z)
ax2.plot(u2,z2)
ax2.errorbar(u3,z3,0,e,color = 'black',linestyle = 'none')
ax2.scatter(u3,z3,35,color = 'purple')
ax2.axis([-40,120,-20000,0])
ax2.xaxis.tick_top()
ax1.set_xlabel('veloctiy (cm/s) at 150 W')
ax1.set_ylabel('depth (cm)')
ax2.set_xlabel('veloctiy (cm/s) at 110 W')
plt.title('Plot of Zonal Velocity against Depth at 150 W and 110 W
            Including TAO/TRITON Data',y = 1.08, x = -0.3)
ax2.legend(labels = ["Low Resolution","High Resolution","TAO/TRITON Data"],
                    loc = "lower right",ncol=1)
ax2.axvline(x=0,color = 'grey')
ax1.axvline(x=0,color = 'grey')
return plt.show()
```

APPENDIX F
PYTHON CODE USED TO FIND TEMPERATURE OF MAXIMAL ZONAL VELOCITIES AT LOW AND HIGH RESOLUTIONS ALONG THE EQUATOR BETWEEN 140°W AND 110°W

The following code was used to generate a line plot of temperature of peak zonal velocities between 140°W and 110°W. In order to run this code, the followign modules must be imported:

```
import numpy as np
from netCDF4 import Dataset
import matplotlib.pyplot as plt
from matplotlib import pylab
```

```
def maxtemp_reg():
    nc = Dataset('ctrl.g.e11.G.T62_gx1v6.002.pop.h.0400.annualmean.nc','r')
    temp = []
    long = []
    for i in range(221,257):
        vel = nc.variables['UVEL'][0,5:20,184,i]
        index = np.argmax(vel)+5
        long += [nc.variables['ULONG'][184,i]]
        temp += [nc.variables['TEMP'][0,index,184,i]]
    nc.close()
    plt.figure()
    pylab.xlabel('Longitude ( E )')
    pylab.ylabel('Temperature ( C )')
    plt.title('Modelled Temperature of Zonal Velocity Maxima')
    m,b = pylab.polyfit(long,temp,1)
    plt.plot(long,temp)
    long = np.array(long)
    print("LR:")
    print("average = ",np.mean(temp))
    print("variance = ",np.var(temp))
    print("std deviation = ",np.std(temp))
    print(m," ",b)
    nc = Dataset('ctrl.g.e11.G.T62_t12.002.pop.h.UVEL.0026.annualmean.nc','r')
    nc2 = Dataset('ctrl.g.e11.G.T62_t12.002.pop.h.TEMP.0026.annualmean.nc','r')
    temp = []
    long = []
    for i in range(3199,3599):
        vel = nc.variables['UVEL'][0,5:20,1181,i]
        index = np.argmax(vel)+5
        long += [nc.variables['ULONG'][1181,i]]
        temp += [nc2.variables['TEMP'][0,index,1181,i]]
    nc.close()
    nc2.close()
    long = (np.array(long)+360).tolist()
    plt.plot(long,temp)
    m,b = pylab.polyfit(long,temp,1)
    long = np.array(long)
    plt.legend(labels = ["Low Resolution","High Resolution"])
    print("\nHR:")
    print("average = ",np.mean(temp))
    print("variance = ",np.var(temp))
    print("std deviation = ",np.std(temp))
    print(m," ",b)
    return plt.show()
```

Appendix G
Larger Displays of Quiver Plots

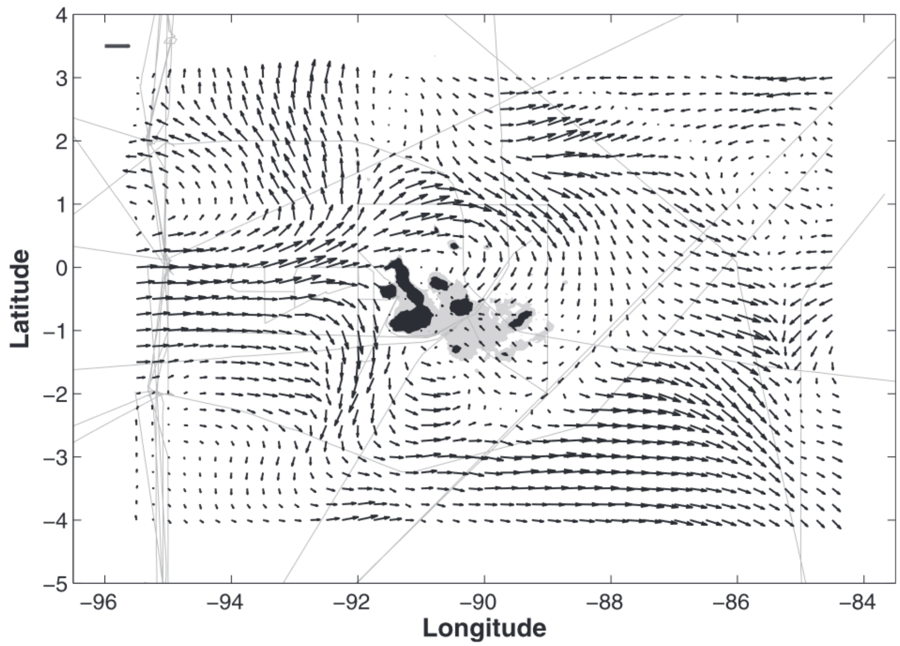


Fig. 12. Larger Display of Karnauskas et al. Observing the GalapagosEUC Interaction: Insights and Challenges (2010), Figure 5; figure 2 in this paper.

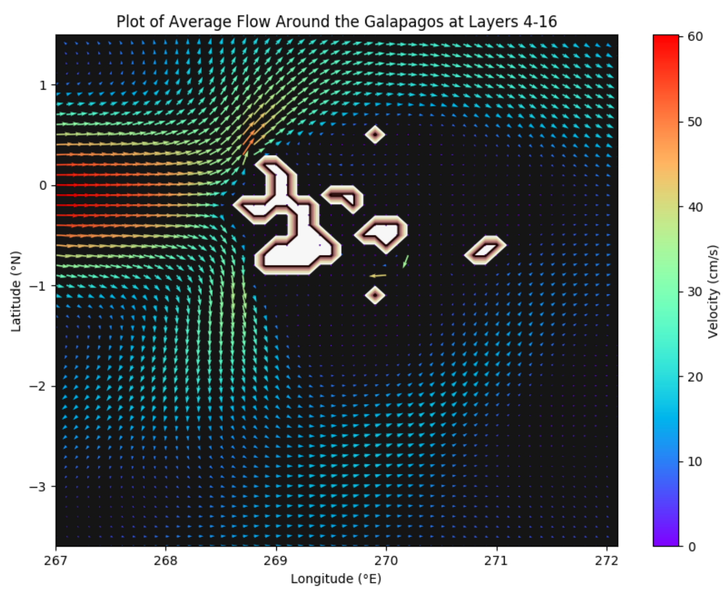
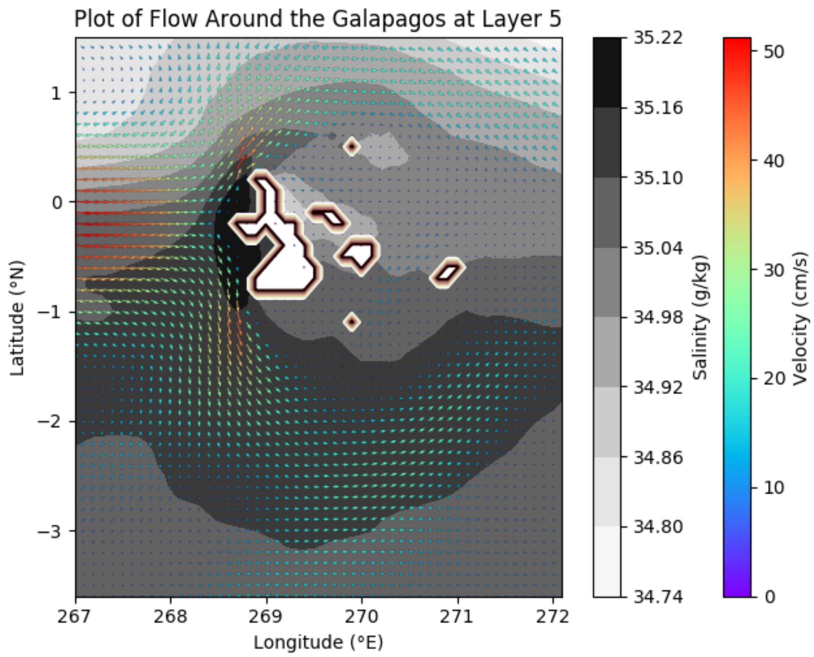
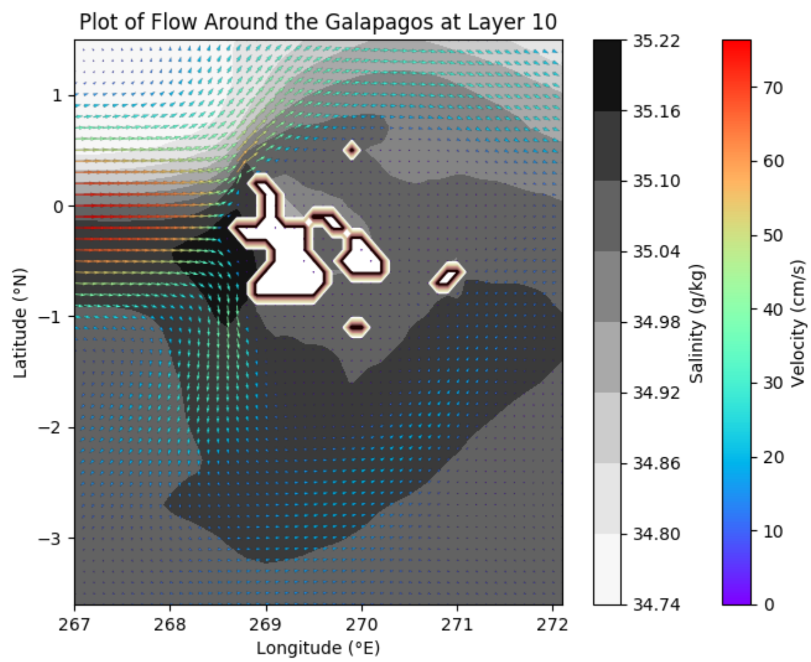


Fig. 13. Larger Display of depth-averaged quiver plot of high-resolution POP2 model; figure 6 in this paper.


Plot of Flow Around the Galapagos at Layer 5
Latitude (°N)
Longitude (°E)
Salinity (g/kg)
Velocity (cm/s)
1
0
−1
−2
−3
267
268
269
270
271
272
35.22
35.16
35.10
35.04
34.98
34.92
34.86
34.80
34.74
50
40
30
20
10
0


Plot of Flow Around the Galapagos at Layer 10
Latitude (°N)
Longitude (°E)
Salinity (g/kg)
Velocity (cm/s)
1
0
−1
−2
−3
267
268
269
270
271
272
35.22
35.16
35.10
35.04
34.98
34.92
34.86
34.80
34.74
70
60
50
40
30
20
10
0

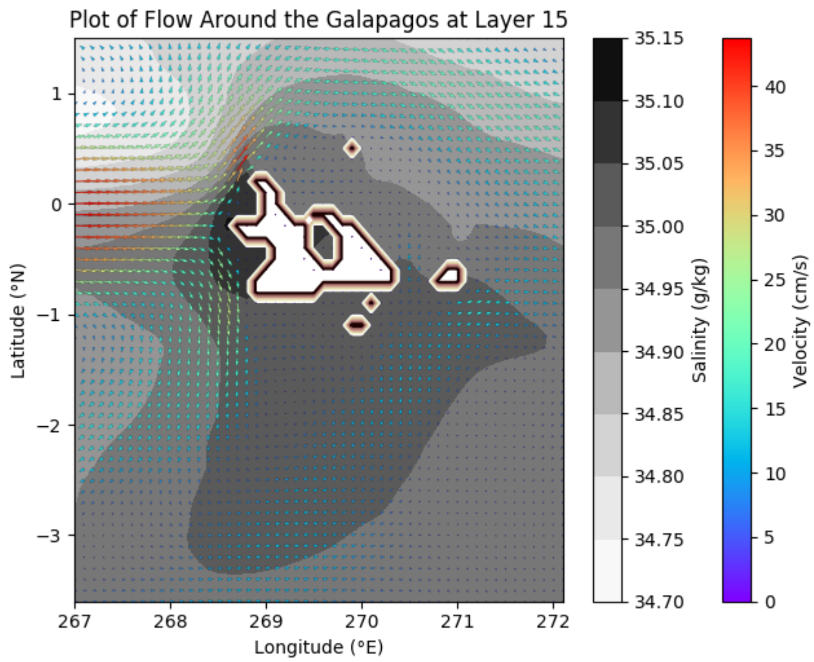


Fig. 14. Larger Display of quiver plot of high-resolution POP2 model at the 5th, 10th and 15th layers; figure 7 in this paper.

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I would like to acknowledge also OCS Project Office of NOAA/PMEL for their providing the relevant data from the TAO/TRITON array.

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