FACULTY OF SCIENCE UNIVERSITY OF COPENHAGEN



Bachelor's thesis

Measuring the Superconductor-to-Insulator Transition in a Semiconductor Josephson Junction Array

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Abstract

In this thesis I present a new system in which the superconductor-to-insultator transition (SIT) can be studied. The system consists of a well defined array of superconducting islands coupled via proximity effect (Josephson junction array). The array is chemically etched in a square pattern of a thin aluminum (Al) film, epitaxially grown on a superconductor/semiconductor hybrid material. The advantage of this new system is related to the semiconducting properties maintained in the system, hence the carrier density can now be controlled by gate-tuning. This provide a knob of E_C/E_J to drive the system through the SIT.

I present a study of different tuning parameters, including gate voltage, perpendicular field and temperature and I provide with a complete phase diagram for a semiconductor Josephson junction array and show behavior similar to a 2D disordered superconducting thin films. The well defined periodic structure of the array lead to unique commesurability effects when a perpendicular field is applied. I present a current-driven transition from a frustration dip to a frustration peak. Scaling analysis revealed critical exponents found for integer and fractional frustration fields.

The thin Al film, with the thickness $d \ll \xi_0$, allow for in-plane field studies where the Zeeman energy will suppress superconductivity. The effect of an in-plane field was observed in a gate-temperature driven phase transition, where different values of an applied in-plane magnetic field lead to two main observations; the in-plane field destroy an intermediate metallic state observed in a zero field study and improved scaling analysis with a set of critical exponents diverging as a function of the applied in-plane field.

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Chapter 1

Introduction

A rather intriguing phenomen arise in some materials at low temperature where repulsive electrons produce paired up states, which is more favorable than staying apart. The pairing of electrons leads to a zero resistance state, i.e. perfect conduction, or more referred to as *Superconductivity*.

After is was first discovered in 1911, it took almost half a century before it was understood in a theory developed by Bardeen, Cooper and Schreiffer, celebrated in their unified BCS theory, explaining the underlying physics that would turn almost half of the periodic table into a perfect conductor. However, superconductivity is rather fragile and usually disappears within a few degrees above absolute zero[1]. The superconducting ground state consisting of bound electrons in pairs coupled together via lattice vibrations (phonons), therefore lead to the questions what external effects destroy the pairing and what is the underlying mechanims? This opened up a branch involving superconductivity and the transition into new states of matter. The field has remained active for many decades and still hold up the condensed matter society [2]. In particular when we confine our selves to two dimensions it becomes truly interesting with even more unresolved questions: What external effects can disrupt two-dimensional (2D) superconductivity? How does disorder affect 2D superconductivity and how can we obtain an insulating state in a continuous phase transition; the superconductor-to-insulator transition (SIT)? What new states of matter are there yet to be explored? Many different systems have entered this field and taken on the task to try and resolve some of these questions. Thin films have provided with many interesting and groundbreaking results, studied with either amorphus or granular structure [3] and later 2D phase transitions were reported in oxide layers [4] and in networks of superconductors (Josephson junction array) [5].

Structure of the thesis

This thesis will present an experiment towards exploring the breakdown of the superconducting ground state into an insulating state. I will introduce a new system referred to as a semiconductor Josephson junction array (JJA), providing a platform for studying the SIT and other fascinating properties related to Josephson junction arrays (JJAs), revealed ones the sample was happily cooled down to ~ 20 mK in a dilution refrigerator.

For the rest of the introductory chapter, I will present some background information on 2D superconductivity and the effect of introducing disorder, which lead to different mechanisms behind the breakdown of the superconducting ground state into new phases. In particular, I will focus on the transition into an insulating state reviewed as a quantum phase transition where scaling theory can be applied. The scaling relation will later be used to characterize the SIT observed in the 2D proximity coupled JJA. In Chapter 2, I will provide a more detailed description of the sample where quantum phase transitions can be studied. This will imply a description of a new superconductor/semiconductor hybrid material and a design of the sample, utilizing some of the properties this material provides. The semiconducting properties which are still remained in the system, gives a new knob where the carrier density can easily be controlled electrostatically, and will constitute as a tuning parameter in the study of SIT. I end this chapter with a few measurement techniques involved with low temperature physcis and details on using a dilution refrigerator. In Chapter 3, I will present some resluts found in the study of this new system. The chapter serves as an elaborating part and should be combined with Appendix A, where the main results of the study will be presented as a first draft manuscript in a paper format. In the chapter I provide a detailed description of the method used to perform scaling analysis to extract critical exponent related to the characteristic phase transition. In addition to, I present some more detailed results discussed in the manuscript. I therefore recommend the reader to jump to Appendix A before moving on to Chapter 3. Chapter 4 will summarize the study and comment on some results, leading to a discussion of open questions and further analyses to be made.

Appendix A presents the experiment and main results in a paper form with purpose of publishing after further analysis have been performed. Appendix B,details on the fabrication procedure for the devices made on an InGaAs/InAs heterostructure. Appendix C, present in a table form main results of two different measured devices (A and B), showing similar characteristic phase transitions. Appendix D show two supplental figures.

To clarify figure reference; figures with decimal numbering refer to chapters of the thesis part, while none decimal numbering are used for figure reference in the manuscript, see Appendix A.

1.1 Superconductivity in two-dimensions

So why are two-dimensions so much different form having three dimensions and why have so many scientists devoted them selves to study this particular low dimensional system? The explanation can be parted into two, where the first is related to the effect of disorder on superconductivity. It seems rather intriguing why superconductivity should still exist, since 2D is the marginal dimension of localization and superconductivity[2], hence in 2D an interplay between the two effects bring forward the question why superconductivity should even exist? The second part is related to the underlying mechanism that breaks down superconductivity in two-dimensions and the entry to new states of matter.

Disorder and superconductivity

Superconductivity is well understood with the highly developed BCS theory attributing the superconductivity to pairing electrons (Cooper pairs), held together in a many-body phase-coherent state, with a binding energy defined to be 2Δ , where Δ is the gap[1]. We can characterize the superconducting state by a complex order parameter $\Delta(\mathbf{r}) = |\Delta(\mathbf{r})| \exp(i\phi(\mathbf{r}))$, and surpression of $\Delta(\mathbf{r})$ to zero, hence destroys the superconducting state. We now gradually introduce disorder to the system; in the first limit of weak disorder Anderson argued in his study on localization [6] that weak disorder is not enough to destroy pairing correlations, hence superconductivity is unaffected in this limit. Even in the limit where all single particle eigenstates become localized superconductivity still exists [2]. By further increasing disorder we eventually approach the critical regime where all electrons are localized on single sites and we obtain a superconductor-to-insulator transition (SIT).

The fermionic and bosonic mechanism

The interplay of localization and superconductivity brought forward a rich field of studying the nature of the SIT and still adresses many open questions. Two competing models try to explain the mechanism that drives the system into a new phase. First, it was argued that a combined action of Coulomb interactions and disorder, would reduce the pairing amplitude and break the Cooper pairs up into their consistutent fermions *fermionic mechanism*.

Another side holds that Cooper pairs would remain bound and view the SIT as caused by phase fluctuations, which destroy long-range phase coherence. This idea was modelled by Fisher in 1990[7], where increasing disorder for charge-2e bosons showed enhanced phase fluctuations *bosonic* mechanism. In his study he included temperature- and magnetic-field-driven SIT and provided

with a complete phase diagram for 2D disordered superconductors[7]. It became an important milestone for the field, where he predicts the SIT to happen at exactly the universal quantum pair resistance $R_Q = h/(2e)^2$.

1.2 The SIT - A quantum Phase transition

The SIT is an example of a quantum phase transition (QPTs) where the system is driven through its critical point by the change of some control parameter. The phase transition is a zero-temperature phase transition, however, it also affects the behavior of a system at finite temperature [8]. In the vicinity of the quantum critical point between the two phases, there are many competing interactions, however a slight change in the control parameter will favour one type of order over the other. These fluctuations between the two states are quantum fluctuations and is the driving force of the QPT [9].

The transition explained can be pointed to different control parameters, depending on the system that are being driven into the insulating phase. In 2D thin films changing the parallel- or magnetic field, disorder, charge carriers, etc. can bring the system through the critical point [3, 4, 5] also reported in some JJAs [10] where an insulating state has been observed. In more recent studies a transition to a metallic phase was found [11, 5].

For a general view of the QPT controlled by a parameter δ , a generic phase diagram [2] is presented in Fig. 1.1. The two phases; superconducting (S) and insulating (I) phases are viewed on each side of the quantum critical point δ_c . By increasing the control parameter δ , it changes the superconducting transition temperature T_c , and vanishes completely at δ_c before entering the insulating part of the phase diagram. In the middle where the system is neither superconducting nor insulating we define the quantum critical (QC) region where quantum fluctuations are enhanced and QC scaling is obeyed. This might be the open window for experimental work to figure out what happens in the shadowed fan of QC behavior.



Figure 1.1: Generic phase diagram of a SIT presented as quantum phase transition. SIT controlled by a parameter δ cross from superconducting state to an insulating state. The quantum critical point δ_c is where the system enter the insulating part of the phase diagram. In the middle where the system is neither superconducting nor insulating there is quantum critical region.

Scaling model near the critical region

In this section a scaling model will be presented and later used in the study of SIT in a 2D proximity coupled Josephson junction array. The theory of quantum phase transitions is constructed in a similar way as thermodynamic phase transitions, and according to the scaling hypothesis[8] all physical quantities near the vicinity of classical phase transitions have a singular part, which shows power law dependencen on a variable with the length dimension and leads to divergent lengths near a phase transition. The correlation length ξ depends on the proximity to the phase transition, determined by the value of the control parameter, eg. temperature, hence the correlation length is defined [9]

$$\xi = \xi_0 \mid t \mid^{-\nu}, \tag{1.2.1}$$

and diverges at the transition point. The exponent ν is the correlation length exponent, ξ_0 is zero-temperature correlation length and $t = |T - T_c|/T_c$ is the reduced temperature. Apart from the correlation length I introduce the relaxation time of the order parameter in the critical region

$$\tau \sim \xi^z, \tag{1.2.2}$$

and z is the dynamical critical exponent. If the transition is controlled by some other parameter, δ , than temperature, eg. magnetic field or disorder, it simply changes t and I define the general expression

$$\tau \sim |\delta|^{-z\nu} . \tag{1.2.3}$$

Divergent correlation lengths and times with divergent behavior of the mesureable quantities near the phase transition lead to the use of scaling analysis, which reveal critical exponents. The scaling theory can further be used to determine the universality class of the transition [9, 2]. Matthew Fisher [2] presented a scaling theory at nonzero temperatures, to be used to characterize the measured resistance in the regime of critical fluctuations. Written in terms of control parameter δ and temperature this leads to

$$R(\delta, T) = R_c \mathcal{F}(|\delta - \delta_c| T^{1/-z\nu}), \qquad (1.2.4)$$

where R is the sheet resistance, R_c is the resistance value at the critical point and \mathcal{F} is an arbitrary function. The scaling relation assumes a continuous and direct SI transition controlled by some parameter. In percolation theory it suggests two distinct numbers to classify whether the system obey classical or quantum effects. For classical percolation $z\nu = 4/3$ and for quantum percolation $z\nu = 7/3$ [12]. When it comes to experimental use of (3) we measure the product $z\nu$, however, for a system with long-range interactions, like a bosonic system where Coulomb interactions are longerange, z is believed to be unity, leaving ν as the variable exponent to classify the transition [2]. The scaling of measured resistance will be used to characterize a SI transition of the 2D proximity coupled Josephson junction array reported in this thesis.

1.3 JJA - A platform to study SIT

Through years different systems have entered the field of studying the superconductor-to-insulator transition can be studied and provided new insights in 2D superconducting systems; do we obtain a direct transition to from a



Figure 1.2: Schematic of a Josephson junction array of square superconducting islands. Josephson junction array of superconducting islands weakly coupled by tunnel barrier. The junctions can be characterized by the Josephson coupling E_J and the junction capacitance E_C . Each island has a self-capacitance C_0 to the ground far away.

superconducting to insulating state and what underlying physics determine the transition?. In this thesis I will present a system providing a new platform in which the SIT can be studied. This involves a combination of a new superconductor/semiconductor hybrid material and a well defined 2D array of superconductors. I will in this section breifly state a model of an array of superconductors, more familiar known as a Josephson junction array.

The array can be thought of as a network of superconducting islands that are weakly coupled by a tunnel junction, shown in Fig. 1.2. The tunnel junction is characterized by the coupling strength between the adjacent islands, determined by the Josephson energy $E_J = \Phi_0 I_c/(2\pi)$, with I_c as being the junction critical current[13]. The junction has a capacitance C used for determining the charging energy $E_C = e^2/2C$. Each island is coupled to a ground far away and has a self-capacitance of C_0 . These are the two characteristic energy scales; where the Josephson energy, associated with tunneling of Cooper pairs between the superconducting islands, and the charging energy, setting the energy scale for which one extra electron charge can be added to a neutral island. The competition between the single-electron effects and the Josephson effect gives rise to SIT when the ratio E_C/E_J is varied [13].

Josephson junction arrays also provide an ideal system to study frustration effects when a magnetic field is applied perpendicular to the array. This is unique for a periodic array of superconducting islands and is where a JJA profoundly differ from its cousin; granular superconducting thin films. In granular thin films the small grains constitute as small superconducting islands and are randomly spread [8]. They have different sizes and various coupling energies. In a well defined periodic array there is a controlled way to continously tune the coupling strength and the periodic lattice structure will lead to phase frustration effects, and will be discussed in later in the this thesis.

Chapter 2

Device design and measurement setup

The design of the sample used for studying the SIT in a Josephson junction array will be presented before moving on to the measurement setup. For detailed fabrication steps I refer to Appendix B, where each fabrication has been divide in the different sections:

- \triangleright Mesa patterning
- \triangleright Island patterning and etching
- > Deposition of insulator
- \triangleright Deposition of top gates

2.1 Design of SIT sample

The design of the sample used to study the SIT and material properties will be presented in this section. The SIT sample has been cleaved in dimensions $2.5 \times 5 \text{ mm}^2$ of wafer JS118, which utilizes an InAs/GaAs quantum well structure with 7nm epitaxial grown aluminum (Al) on top, see layer stack of JS118 in Fig 2.1. The wafer has been grown in Chris Palmstroms laboratory in Santa Barbara, CA, by Javad Shabani, and was carefully characterized by the 2DEG team of Center for Quantum Devices with reference to Morten Kjærgaard's Thesis [14], in which details upon mobility and density measurements can be found along with properties of the thin Al film. Using a Hall bar geometry the mobility and 2DEG electron density of the wafer was found to be $\mu = 7.500 \text{ cm}^2/\text{Vs}$ and $n = 3.2 \cdot 10^{16} \text{m}^{-2}$ respectively. These measurements were performed at zero gate voltage where the system has two subbands. To avoid the two subbands I operate at a slightly negative voltage, where the system is believed to be in the single band limit.

The device design consists of an etched mesa patterned in a Hall bar convetion, with Al arrays (grey) constituting as the Josephson junction arrays. The array of 40x100 islands has been made by chemical etching the thin Al film into a square lattice, leaving the 2DEG (blue) exposed between each superconducting island, shown in Fig. 2.1a. The array is covered witg a topgate (gold), constituting as anew knob to control E_C/E_J . Between a topgate and the Al array a dielectric layer of Al_2O_3 oxide layer has been deposited.

The sample is designed to utilize coventional Hall measurements, hence probing the longitudinal voltage V_{xx} and the Hall voltage V_{xy} . V_{xy} data will not be presented in this thesis. In order to perform 4-terminal measurement each Hall bar is designed with 4 side probe contacts and 2 end contacts constituting source and drain. The complete sample consists of a total of 12 Hall bars, each with a different array pattern characterized with the spacing between the aluminum islands, a and the dimension parameter of the square island, b, see Fig. 2.1b. The Hall bars each have a length of 150μ m and a width of 40μ m, where the length is taken as the distance between the



Figure 2.1: Semiconductor/superconductor Josephson junction array. a Schematics of the device, involving a square array of superconducting Al islands. The Al film is grown epitaxially on a InGaAs/InAs heterostructure c. The device is connected to a current source and measured using a four-terminal measurement for the longitudinal resistance V_{xx} . b Scanning electron micrograph of the sample. False colored us to map the layers onto c where the layer stack of the material is shown.

two end contacts. This size of the Hall was chosen for convenience in the fabrication steps of the sample. The size of the array pattern exactly fits into one writing field determined by the provided e-beam system, and will preventing stitching errors once the array is patterned on each Hall bar. Stitching errors will create irregularieties in the array pattern, hence destroy the periodicity of the array and is therefore highly prioritized to be avoided in the fabrication of the SIT sample.

The array pattern has been extended on both sides of the side probes on the Hall bar, however, the probed part of the array will be set by the spacing of the side contacts, $L = 110 \mu m$ and the width of the array, which is equal to the Hall bar width, $W = 40 \mu m$. The ratio L/W is used to define the resistance per square R_s . Further details on array patterning will be specified in the fabrication steps in Appendix B.

2.2 Measurement setup

Dilution refrigerator

The measurements presented in this thesis has been performed in a Triton cryofree dilution refrigerator, providing the cooling source for the experiment to reach low enough temperatures to obtain a superconducting state in the thin Al film. The unpatterned Al film has a critical temperature of ~ 1.6 K [14]. To study the superconductor-to-insulator transition lower temperatures are prefered to enter the regime of quantum critical behavior. The fridge can provide an extensive cooling process by mixing ³He and ⁴He in the mixing chamber unit and by continously distrubing the equilibrum phase, the fridge can reach a base temperature of $T_{mc} = 20$ mK, providing the right environment where k_BT is low enough to no longer hide quantum effects. The measurements have been performed in a newly setup fridge system. In the setup of the fridge, RF and RC PCB filters were installed. The RC filter is a three stage of first a 7-pole π filter with a cut-off frequency $f_c = 80$ MHz. The next two stages are low pass filters with $R_2 = 500 \Omega$, $C_2 = 2200$ pF and $R_3 = 1200 \Omega$, $C_3 = 1000$ pF. In the cooldown of the first sample, the resistors in the RC filters was found to become superconducting. The resistors was made with a Tantalum Nitride Resistive film, which turned superconducting ~ 1.5 K. In the next cooldown the RC filters had been change to filters with non-superconducting resistors. However, no significant changes was found in the measurements of the two cool downs.

The sample has been mounted on a Copenhagen board sample board, which has been developed for RF resonators and provide 48 DC lines. For the SIT experiment I will operate at such low frequencies < 100 Hz that fast lines will not be needed. The sample was bonded, using a wire bonder of silver thread and then mounted in the puck according to the vector magnet installed in the fridge. The magnet is (1,1,6) T vector magnet and the sample has been placed in the orientation that allow for the highest field in the plane of sample when loaded in the dilution refrigerator, see Fig 2.2a.

Measurement teqniques

I here present a typical setup for the measurements performed on the SIT sample. I will describe some of the main strategies to account for when measuring in a voltage bias setup and in a current bias setup. In both cases I have used a 4-terminal setup.

For the voltage-bias setup, I source $V_{in}^{AC} = 5 \ \mu V$ by using a Stanford Research SR830 Lock-in, providing the AC signal at a frequency f = 77 Hz through a voltage divider with a factor 1/10.000. The AC excitation is low enough to ensure not smearing out important features in the measurements. By increasing the excitation the signal strength will improve but also introduce heating of the sample. I use a Low Noise/ High Stability IV Converter SP983 IV to measure the current through the sample V_{out}^{AC} . I probe the longitudinal voltage V_{xx} using a home build preamplifier with a sufficiently high input impedance to allow measurements in the insulating regime where the resistance is of the order $R \approx 100 M\Omega$. I control the voltage on the topgate using a DAC, providing a range of ± 10 V. The described setup of a voltage bias measurement is shown in a schematic in Fig. 2.2b. The magnetic field was applied using two different methods; the Oxford Power Supply is used to power the magnet when sweeps in the range of 1T is needed, for example in in-plane field studies the where the critical field of the Al is ~ 1.6 T. In perpendicular field studies much lower field strengths are required, < 100 mT, and very fine sweeps at low field was made using a combination of standard 2400 Keithley Source meter and a Kepco Power Supply to amplify the current. A magnetic field strength of 100 mT in an Oxford standard vector magnet require a current of 5.9 nA, and cannot be provided by a standard Keithley. The Kepco has a DC output range of ± 20 A.

A different setup is used for current bias measurements. A Yokogawa provide as the DC voltage supply through a 100k Ω bias resistor. When the sample resistance is of the order of the bias resistor, some of the current will start to flow back and only a percentage through the sample. It is therefore important to measure the current that actually goes through. The critical current of which the array becomes normal is $I_c^A \approx 20 \,\mu\text{A}$, which sets the current scale for the DC measurements. I bias with an AC component through 1 G Ω resistor with 5V excitation, which gives $I_{AC} = 5$ nA. I measure the differential voltage through a voltage amplifier that amplifies both AC and DC components of the signal by a factor of 100. The output is measured with a lock-in for the AC part and a DMM to measure the DC component. The differential resistance, dV/dI is found from the measured AC voltage and current. The resistance R is found from dividing the measured DC voltage with current. а





Figure 2.2: **Measurement setup**. a Orientation of sample JS118.12 bonded on Copenhagen board in the puck. b Voltage bias setup, showing 4-terminal measurement of the device. Source $V_{in}^{AC} = 5 \mu V$ by using Lock-in, providing the AC signal at a frequency f = 77 Hz through a voltage divider. The current is measured through IV converter and I probe the longitudinal voltage V_{xx} using a home build preamplifier, which amplies by a factor of 1000. The AC signal is measure with a lock-in. c Current bias setup, showing 4-terminal measurement of the device. The DC voltage supply through a 100 kΩ bias resistor combinated with 1 GΩ resistor with 5V excitation, leading to $I_{AC} = 5$ nA. I measure the differential voltage through a voltage amplifier that amplifies both AC and DC components of the signal by a factor of 100. The output is measured with a lock-in for the AC part and a DMM to measure the DC component.

Chapter 3

SIT and commensurability effects in a semiconductor JJA

In this chapter I will present data that serves as supplement to the paper in Appendix A. In the chapter I will provide with another version of the three parameter phase diagram to highlight a few features explained in the manuscript and the observed transitions in relation to the boson model, first formulated by Matthew Fisher in 1990[2]. I will present commensurability effects, including low field frustration effects and vortex penetration, observed at higher fields for two different gate voltage values values. I end this chapter with a detailed description of a method used to perform a scaling analysis of the vortex insulator-to-dynamic votex transition.

3.1 Phase diagram

In this section I present the phase diagram discussed in Appendix A with a few more remarks and details in relation to the different transitions dependent on the tuning parameter. I discuss the phase diagram according to the dirty boson model of 2D superconducting thin films proposed by Matthew Fisher[7]. At finite field the semiconductor Josephson junction array exhibit a superconducting state, which will be destroyed ones the temperature, T is increased or the gate voltage, V_{TG} , is decreased. A gate-temperature driven SIT is shown in ground plane in Fig. 3.1a.

According to the boson model (Fig. 3.1b), the disorder-temperature tuned phase transition can be described in terms of vortex unbinding, in the same way as a Berezinskii–Kosterlitz– Thouless (BKT) transition, where vortex are paired in the superconducting state and unbind when entering the insulating state. It is expected that a Josephson junction array undergo a BKT transition and has been observed in different experiments on JJAs [13, 15]. It is therefore favorable to study the gate-temperature driven phase transition captured for this new type of JJA system, to see whether the system undergoes a BKT transition. A model that encounter the structure of the square Al array is therefore needed to determine if there is a BKT transition. In ref. [3] a method to determine the BKT transistion temperature is found by fitting the BKT square-root dependence on tempererature [13]

$$R_0(T)/R_N = c \exp(-b[E_J/(T-T_J)]^{1/2}), \qquad (3.1.1)$$

where b and c are constants of 1st order, the normalized temperature is defined $T/E_J(T)$. The resistance R_0 is normalized with respect to the normal-state junction resistance R_N . The model presented in (3.1) can be a way to analyze the gate-temperature tuned transition and determine if there is a BKT transition, also predicted to occur in the disorder-temperature driven transition in the boson model.

At zero-temperature and at finite magnetic field we move along the V_{TG} axis, (left side plane of Fig. 3.1a). The system exhibit a true superconducting state and upon increasing the magnetic field or V_{TG} superconductivity will be destroyed and the system enter and insulating state is entered. In the dirty boson picture this is described as a vortex-glass phase, where Cooper-pairs



Figure 3.1: **Phase diagram** Three parameter phase diagram of a semiconductor/supercondutor Josephson junction array. Phase diagram for three control parameters V_{TG}, B_{\perp} and T for a semiconductor Josephson junction array. Contour lines of real data sets with 100 Ω and 250 Ω highlight the concavity in the thermodynamic phase plane at $V_{TG} = -3$ V. At contour line at 6.45 k is plotted to show the regime where resistance is comparable to the quantum pair resistance. **Model of 2D dirty superconductor** Schematic phase diagram for a 2D disordered superconducting

are localized while vortices condense ones we enter the insulating phase. In the superconducting state the opposite happens where Cooper-pairs have condensed (bose condensed) and vortices are localized. In this scenario the Cooper-pairs are present in both phases, and therefore the quantum pair resistance $R_Q = h/(2e)^2 \approx 6.45 \text{k}\Omega$ is a universal value for the transition. In Fig. 3.1a, a green contour line corresponding to $6.45 \text{k}\Omega$ has been plotted in the phase diagram for the JJA. Both the gate-field tuned and gate-temperature tuned SIT do not transition at the universal value. There might be different explanations for this observation and leads to the question whether Cooper-pairs have been broken at the transition and system is now dominated by fermionic physics?

The thermodynamic phase plane, the temperature-field driven transition, has a very different concavity compared to the diry boson model. This side of the phase diagram has been studied extensively in a current-voltage setup and I provide a detailed description in Appendix A.

3.2 Commensurability effects in a JJA

The contructed Josephson junction array has been studied intensively in a perpendicular magnetic field where magnificent periodic features emerge. This is where JJAs stand out from studies in thin films and granular structures where these effects will not be present. The well defined periodic structure in a JJA gives rise to low field frustration effects. Vortices enter the array above a threshold and due to the junction arrays periodic lattice struture, it gives rise to a potential where vortices are pinned to the sides. Only above the depinning current vortices will start to flow[16]. And at low magnetic fields the properties of the array are determined by the single vortices entering the array and upon increasing the magnetic field more commensurability effects come into play [8, 1].

The vortices enter the array in a rational number f = p/q and the groundstate of the system consists of a cherckerboard configuration of vortices with a $q \times q$ elementary cell[1]. For f = 1/2the vortices are filling half of the array and for the integer f = 1, one vortex has filled each elementary cell, shown in Fig. 3.2a. When the array is in one of these groundstate configurations, it resides in a superconducting state, represented as minima in R(B), see Fig. 3.2b, where a collection of integer frustration minima at low field values is observed. For a square lattice minima are expected to appear at f = 1/2, 1/3, 1/4, 2/5, ..., and small changes in the magnetic field can disrupt the commensurated state so vortices become mobile, hence lead to dissipation [8], which will accompain the flow of current through the lattice junction.

The first integer frustration minima is expected to appear at $B_0 = \Phi_0/A$, with $A = (a + b)^2$ being the area of the unit cell and Φ_0 is the flux quanta. Frustration minima was observed in two



Figure 3.2: Commensurability effects in a Josephson junction array. a In a Josephson junction array vortices can enter the array in a rational filling f = p/q and the ground state consists of a checkerboard configuration of vortices. Here showed example for f = 0, 1/2 and 1. b Commensurability effects observed in sample JS118.12. Reveal low field frustration effects and at high field vortex penetration effects appear.



Figure 3.3: Commensurability effects at different gate voltage values. a $V_{TG} = -3$ V, show both low field commensurability effects and high field features related to vortex penetration of the superconducting Al islands. b $V_{TG} = -3.65$ V, low field commensurability effects are smeared out while vortex penetration effects are more pronounced.

samples; device A with a separation of b = 150 nm and device B with a separation b = 350 nm. The area of the unit cell will be slightly different in the two samples. For device A, the first minima

corresponding to interger f, is expected to appear at ~ 1.8 mT, and the next at minima $2B_0$, and so forth. In Fig. 3.2b clear frustration dips of device A appear with a period of ~ 1.7 mT ending with the last observed minima at ~ 7 mT, corresponding to the 4th frustration field where the ground state consists of 4 vortices aligned in each elementary cell. Dips at fractional field frustration values appear in between the integers. Each trace in Fig. 3.2b correspond to different temperatures, where blue is lowest and red is highest. By continously increasing the temperature the frustration minima smear out and eventually disappear at ~ 1.5 K.

In reference to Fig. 3.2b, at relatively higher field a different but strong periodic feature appear with the first event at $\sim 12 \text{ mT}$ at low temperature, persisting even at high temperatures where the event has moved down in field. This effect is not related to the alignment of vortices in the periodic lattice, instead it reflects the shape of the superconducting island. The feature is the vortex penetration field of the superconducting island and the first event correspond to exactly one vortex penetrating the islands. For each $\sim 4 \text{ mT}$, the next event appear, corresponding to the next integer number of vortices pentetrating the island. The effect has been studied in a 1 μ m square aluminum island reported in ref. [17, 18].

The penetration fields are strongly dependent on temperature and decrease uniformly when the temperature is increased. The effect has been observed at different gate voltage values both in the regime where the system obtain its normal state resistance and appear even stronger in a regime where a field-tuned transition to a metallic state is observed at the critical value $B_c = 40 \text{ mT}$.

3.3 Vortex insulator-to-dynamic vortex state

The vortex insulator-to-dynamic vortex state appear as a clear transition from frustration dip to frustration peak in the measured differential resistance dV/dI, see Fig. 3.4a. However, the measured resistance R = V/I remain dips at all frustration values through the transition, see Fig. 3.4b,. One interpretation suggests that the system is still pinned, as the resistance would capture dissipation due to depinning [19] and the transition from frustration dip to a frustration peak must be related to another mechanism prveiously explained as a dynamic Mott transition [19].

The transition can be viewed in the same way as a superconductor-to-insulator transition with current as the control parameter, referred to as a current-driven transition. It is possible to identify the seperatrix that divide the two different states; the insulating vortex state from the dynamic vortex state where frozen vortices have become mobile. In the language of phase transitions this leads to scaling of the measureable quantity with respect to the order parameter, and reveal a critical exponent which characterizes the system near the transition.



Figure 3.4: Frustration effects with an applied electric current . a The measured differential dV/dI resistance show a current-driven transition from frustration dip to frustration peak. b The measured resistance R, remain a dip at frustration filling values when an electric current is applied.

Scaling model of frustration dip-to-peak

In this section I will provide a more detailed decription of a method used to perform scaling analysis in relation to equation (1.2.4). The scaling relation can be written in terms of the measureable quantity dV/dI, the scaling order parameter I and the variable $b = f - f_c$, where f_c is the field frustration value. The scaling relation takes the form [19]

$$\frac{dV(f,I)}{dI} - \left[\frac{dV(f,I)}{dI}\right]_{I=I_0} = \mathcal{F}\left(\frac{\mid I - I_0 \mid}{\mid b \mid^{\varepsilon}}\right).$$
(3.3.1)

I will present the method used to find the critical exponent for transition of frustraion dip-to-peak of f = 1. For this type of transition there is a left and right side transition, to be analyzed independently. I find the separatrix for both sides from the condition $d(dV/dI)/df|_{f=f_c} = 0$, revealing one for each side. The separatrix on left side correspond to a critical current of $I_0^L = 2.5\mu$ A and the right separatix is equal to a critical current of $I_0^R = 2.35\mu$ A. I proceed to analyze the sides independently by first identifying the regions where the transitions are clear. The left and right transition are presented as two subpanels in Fig. 3.4a and appear in regions far from the frustration minima. If we move further into the critical region, the transition obtain some behavior of the other side and scaling is not possible close to f_c .

A log-log plot of $d[dV/dI - dV/dI(I - I_0)]/dI$ and 1/b was constructed for both the left and right side. It's slope is equal to ε if equation (3.3.1) is obeyed. Fig. 3.5b shows the result obtained from the left side transition with both the upper (red) and lower (blue) branch. The points collapse in a region of a same sloped trend until the lower branch saturates while the upper branch continue with a slightly steeper slope. Fitting to the region where both branches collapse, disregarding the outlying points, I obtain a value for the critical exponent for the transition equal to $\varepsilon_L = 1.5$. Following the same procedure for the right side, the two branches are clearly separated and follow different slopes. The upper branch reveal a slope similar to the one obtained from fitting to the left side transition $\varepsilon_R = 1.7$ while the upper branch suggests a larger critical exponent, see Fig. 3.5c. The two different exponents found for the upper and lower branch on the right side, explains why the scaling plot presented in Fig. 5 in the Appendix A is not a good scaling compared to the transition on the left side. This method can be used to determine the critical exponent of more frustration dip-to-peak transitions.



Figure 3.5: Scaling of frustion f = 1 a Transition from a frustration dip-to-peak at frustration filling f = 1. A left and right transition can be studied independently. b Scaling method used to determine the critical exponent for the b left and c right side transition. Upper (red) and lower (blue) are shown in the same plot.

Chapter 4

Conclusion and Outlook

To conclude, the new system of a superconductor/semiconductor Josephson Junction array have successfully demonstrated a gate-tuned phase transition from a superconducting state to an insulating state. In a gate-temperature driven phase transition an intermediate state of metallic behavior was observed and leads to the question if scaling is possible in the regime where of no direct SIT. The data was analyzed according to the relation presented in eq. (1.2.4) and revealed a critical exponent ~ 2.7 . It is still not clear why the scaling works for this transition when the system exhibit an intervening state in SIT.

The gate-temperature driven SIT was studied at different values of an applied in-plane field and revealed a direct transition from a superconducting state to an insulating state, with no intermediate metallic state. A scaling analysis improved scaling and showed a collapse of the critical exponent when the applied field was increased. The diverging exponents are still unresolved, however, seem to clearly depend on the effect of an in-plane magnetic field.

In a perpendicular field commensurability effects at low field values were observed. A currentdriven transition of frustration minima to frustration peak was studied and suggests a transition where frozen vortices become mobile when a critical current is applied. Scaling analysis of the current-driven transition revealed exponents for f = 1 close to 1.5 for the lower branch and 1.7 for the upper branch. A critical exponent of frustration f = 1/2 was found to be ~ 2 . The currentdriven dip-to-peak evolution of frustration fields show same indications of a previously reported vortex Mott insulator to vortex metal transition. The scaling analysis presented here reveal different exponents for each frustration field values, hence the transition presented in this study is still open for interpretation. More frustration field values are yet to be analyzed to see if the critical exponents can divide into integer and fraction frustration filling.

A complete phase diagram of the parameters V_{TG} , B_{\perp} and T, was constructed and show similar transition behavior compared to the previously proposed phase diagram of 2D disordered superconducting thin films [Fisher]. The phase diagram is yet to be examined and an analysis of the gate-temperature driven transition is favored to see if the system undergoes a BKT transition, expected for JJAs[16] and reported in previous studies of JJAs [20].

The thermodynamic phase plane show a different concavity and was characterized in a currentvoltage study, where a field tuned transition to a metallic state was observed and a field-tuned transition to an insulating state was observed in the limit where the normal state resistance is similar to the separatrix $R_x \sim 13 \text{ k}\Omega$.

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Appendices

Appendix A

First draft of manuscript

A Semiconductor Josephson Junction Array

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In the relation to many years attention drawn to two-dimensional (2D) systems of proximitized superconductors and the breakdown of the superconducting ground state by tuning different control parameters, we here report on a superconductor/semiconductor hybrid system combined with a Josephson junction array (JJA), providing a new platform to study the superconductorto-insulator transition (SIT). We present a system with gate-control of E_C/E_J , and study a collection of parameter-tuned transitions, including temperature-, perpendicular magnetic-field- and diorder-tuned transitions from a superconductor/superconductor JJA. We observe a unique set of commensurability effects when imposing a perpendicular magnetic field, and study a currentdriven transition from commensurated field minima to peaks, interpreted in context of a vortex insulating-to-dynamic vortex state. In an in-plane field study, we show a collapse of the critical scaling exponent as we approach the paramagnetic normal state.

Introduction

Through decades 2D superconductors and new phases associated with destroying superconductivity have been studied intensively and still remain an active field. In particular the effect of disorder on superconductivity seemed rather intriguing, why superconductivity should still exist with an interplay between localization and superconductivity [1]. However, experiments have succesfully shown that 2D superconductivity exists and can be destroyed by tuning different parameters, including temperature, disorder and magnetic field. Through years there have been many different approaches towards studying the breakdown of superconductivity in 2D, including granular and amorphous thin films and later studied in Josephson junction arrays and different types of oxide layers [2–4]. The fascination and interst in this field relates to the constant revelation of new unresolved problems associated with the breakdown of the superconducting ground state into new states of matter. In many cases a transition to a metallic state was observed [4, 5] and later other systems yielded a direct transition into an insulating phase where the conductivity exhibits weak localization, strong localiztion or activated behavior [6]. The mechanism behind the superconductor-to-insulator transition (SIT) has pushed competing paradigms either modelling the SIT in terms of phase fluctuations, hence destroying long-range superconductivity [1] (bosonic) or in terms of Coulomb repulsion as the main active ingredient to reduce the pairing amplitude (fermionic).

In the recent years, attention has been devoted to the study of island systems as Josephson junction arrays (JJAs), which in its essense is very similar to a granular superconducting thin film where islands of different sizes are randomly distributed and due to proximity effect leads to a superconducting state. In



Figure 1: Semiconductor/superconductor Josephson junction array. a Schematics of the device, involving a square array of superconducting Al islands. The Al film is grown epitaxially on a InGaAs/InAs heterostructure c. The device is connected to a current source and measured using a four-terminal measurement for the longitudinal resistance V_{xx} . b Scanning electron micrograph of the sample. False colored us to map the layers onto c where the layer stack of the material is shown.

a JJA the ingredients to drive the phase transition is given in the competition between the Josephson energy and Coulomb blockade, which can be tuned



Figure 2: Gate-tuned superconductor-to-insulator transition. a The measure Resistance as a function of temperature. Each curve correspond to a different V_{TG} value. The separatrix to is highlighted with a dashed line at $V_{TG} = -3.75$ V, and correspond to critical value where all temperature cross in a single point in **b** at $R_x \sim 13$ kΩ. **c** Scaling plot of R_s/R_x as a function of the variable t, plotted with best exponent $\alpha = 0.37$ and show data collapsing on both upper and lower branch.

by either physically changing the geometric parameters of the JJA (island size or separation), or by electrostically tuning the density of charge carriers. Approaches towards gate-tuned transitions have been reported, involving specific types of (JJAs) with a hybrid of superconductor-metal-superconductor, where a phase transition into a metallic phase was observed [4, 7].

We study JJA in a new system in which a gatetuned transition to an insulating state was observed. We utilize a new material, consisting of a superconductor/semiconductor hybrid system consisting of InGaAs/InAs 2D electron gas (2DEG) and epitaxial grown Al. Recent measurements reported on gateablility of the material [1] and even showed promising results toward realizing topological states of matter that might be interesting in future studies. These remarkable results encourages to pursue new applications of the material. The advantage of this new system is related to the semiconducting properties maintained in the system, hence the carrier density can now be controlled by gate-tuning, which affects the coupling between islands. We here report on a new gate-tunable system that combines a 2D semiconductor/superconductor with a large Josephson junction array in the sequence super-semi-super.

Our system consists of a proximity-induced array of made out of epitaxial grown aluminum (Al) with a thinkess of 7 nm on top of an InGaAs/InAs heterostructure. The array of 40x100 islands has been made by chemical etching the thin Al film into a square lattice, leaving the 2DEG exposed between each superconducting island (Fig. 1). The islands have a width of $a = 1 \,\mu \text{m}$ and are separated by a distance $b = 150 \pm 9 \text{ nm}$ (device A) and $b = 350 \pm 11$ nm(device B) between the edge of each neighboring islands (Fig. 1.b). Characterization measurements of this material gives a density of $n = 3 \cdot 10^{12} \text{cm}^{-2}$ and a mobility of $\mu = 10^4 \text{cm}^2/\text{Vs}$, yielding a mean free path of $l_e \sim 230 \text{ nm}$ [1], which is comparable to the island distance. These measurements were taken at zero gate voltage where the system has two subbands, we therefore operate at slightly negative voltage, where we believe the system is in the single band limit. The array was covered with a 40 nm of Al_2O_3 consituting as an insulating layer between the array and a topgate (10 nm Ti/250 nm Au) deposited on top of the array (Fig. 1).

The system show that by systematically depleting more and more carriers we observe a transition into an insulating state. We study the system affected by a perpendicular magnetic field in which an intervening metallic phase has been observed along with remarkable commensurability effects reflecting the periodicity of the array and area of each superconducting island. The very thin Al film allow for in-plane field studies, in which the superconducting properties are destroyed and the system enter a normal state at the critical in-plane field value.

We present data on both devices, which have been measured in a dilution refrigerator using four-terminal lock-in measurement setup, unless differently stated we always show the differential resistance dV/dI. We begin by describing measurements of device A and find similar results in device B.

Gate-tuned SIT

We first discuss the gate-tuned transition from superconducting-to-insulating state. In reference to Fig. 2a, we show measurements of the resistance per square islands R_s as a function of temperature ranging from T=30 mK up to 1.8 K. Each curve has a fixed carrier density set by the topgate voltage V_{TG} . We observed a gate-tuned transition from superconductor (decrease of resistance at low T) to an insulating state (increase in resistance at low T) measured up to eight orders of magnitude of R_s . At temperatures ~1.6 K we observe a kink in resistance which persists even at very negative gate values, interpreted as the superconducting transition of the Al islands, consistent with the critical temperature of the Al film [1]. By further cooling the system exhibit a 2D superconducting state for gate voltages above -3.5 V, starting with an abrupt drop in resistance around 1 K at the highest gate value -3 V. We identify curves saturating at finite resistance values as an intervening metallic state where the transition



Figure 3: Three parameter phase diagram of a semiconductor/supercondutor Josephson junction array. Phase diagram for three control parameters V_{TG}, B_{\perp} and T for a semiconductor Josephson junction array. Contour lines of real data sets with 100 Ω and 250 Ω highlight the concavity in the thermodynamic phase plane at $V_{TG} = -3$ V.

temperature is surpressed. Decreasing the gate voltage down to -3.9V we approach an insulating state in which the resistance increases as the temperature is lowered in contrast to the superconducting state. The insulating and superconducting behavior is separated by a curve of constant resistance in the temperature interval, which is denoted the separatix at $V_{TG} = -3.75 V$ (dashed line in Fig. 2a).

Scaling analysis of superconductor-to-insulator transtion—In Fig. 2b we plot R_s as a function of gate voltage with curves corresponding to different temperatures from 30 mK to 1.5 K. They exhibit a single well defined crossing point at a critical resistance value of $R_x \sim 13 \text{ k}\Omega$ at $V_x = -3.75 \text{ V}$ corresponding to the separatrix.

The existence of a crossing point of which the resistance is temperature independent suggests quantum phase transitions (QPTs) where the crossing of the phase boundary changes the quantum mechanical ground state. A scaling theory can be used to characterize the measured resistance in the regime of critical fluctuations where the correlation length ξ and time τ diverges as $\xi \propto |V_{TG} - V_x|^{-\nu}$ and $\tau \propto \xi^z \propto |V_{TG} - V_x|^{-z\nu}$ (assuming a gate-tuned transition), hence determine the critical exponents ν and z in the scaling relation [8]

$$R(V_{TG},T) = R_x F((V_{TG} - V_x)T^{1/z\nu}).$$
 (1)

The scaling analysis was made by selecting curves of temperatures from 60 mK up to 220 mK. By subtracting the separatrix from the measured R_s a scaling analysis with respect to the variable $t = |V_{TG} - V_x| T^{-\alpha}$

was made. In a plot of logarithmic values of R_s vs t, the superconducting and insulating state will develop into two branches, shown in Fig. 2c (upper being the insulating state and lower being the superconduting state). To best represent both branches we find the exponent estimated from finding minima of the variance of slopes extracted from fitting straight lines for different values of the exponent $\alpha = 0.1 - 0.9$ for the upper and lower branch separately in. The best exponent which gave $\alpha \sim 0.37$ yielding a critical exponent of $z\nu \sim 2.70$. The result shown in Fig. 2c resembles previously reported scaling analyses of superconductorto-insulator transistions reporting on different values of the critical exponent ranging from values close to classical percolation $z\nu = 4/3$ [3, 6, 9] to exponents close to quantum percolation with $z\nu = 7/3$, which is often seen for more disordered systems [2]. The best exponent found by fitting to our data is closer to the value of quantum percolation and show scaling up to four orders of magnitude of R_s .

Phase diagram

Historically disorder, T and B are the most studied parameters [10][3] and we here present a phase diagram of the three control parameters V_{TG} , T and B_{\perp} , see Fig. 3. We observed an insulating phase by continously tuning the coupling between the islands controlled by V_{TG} . An insulating phase was obtained for both cases of a gatetemperature and gate-field tuned transition while at the thermodynamic phase plane a field-tuned transition to a normal state was observed when the gate is at the least negative value. The constructed phase diagram shown in Fig. 3 remebles the original phase diagram of the dirty boson model proposed by M. P. A. Fisher [8] and suggests that our superconductor/semiconductor JJ array in many respects behaves as 2D disordered thin films. We see however, a different concavity in the thermodynamic phase plane and we will focus on this part of the phase plane.

We plot contour lines of real data sets with resistance values of 100 Ω and 250 Ω tracing the phase diagram out in space. The contour lines mark an intermediate phase above 100 Ω and below 250 Ω in the thermodynamic phase plane, at $V_{TG} = -3$ V. In this state locally superconducting puddles still exists before the sysem enters a complete normal state. The transition will be explained shortly in a current-voltage study. The contour line of the resistance value 100 Ω traces the temperature dependent critical field $B_c(T)$ while 250 Ω corresponds to $H_{c2} = 60$ mT for $V_{TG} = -3$ V.

Current-voltage characteritics in the thermodynamic phase plane— We zoom in on the field-driven phase transition as a function of temperature and study three planes each fixed at a different gate voltage value. See Fig. 4 starting from left; $V_{TG} = -3V$ (4a),



Figure 4: Thermodynamic phase plane in three regimes of different disorder a, The thermodynamic phase plane for $V_{TG} = -3$ V, with fitted line to highlight the curvature of the feature that dose not depend on gate voltage. Current-voltage characteristic at b T = 30 mK and for c T =800 mK for four different field values. d, Phase plane at $V_{TG} = -3.75$ V where the normal state resistance is similar to the separatrix in Fig. 2. Current-voltage characteristic at four different field values for e T = 100 mK showing a SI transition and f T = 1.3 K, showing a SN transition. g The insulating phase plane with $V_{TG} = -3.85$ V. Current-voltage characteristic at h T = 30 mK and i T = 800 mK show insulating behavior for all field values.

 $V_{TG} = -3.75V$ where the normal state resistance is similar to the separatrix in Fig. 2 (4d) and the more depleted regime at $V_{TG} = -3.85V$ (4g). We notice a feature whose shape does not depend on gate voltage (dashed line). A 4th-order polynomial fuction was fitted in Fig. 4a and replotted in the two more depleted regimes to highlight the curvature of the feature. In order to obtain further insight in the type of transitions in the three different regimes, we studied each in a current-voltage setup.

In the first limit we show dV/dI as a function of $I_{\rm DC}$ for two different temperatures (T = 30 mK and 800 mK)and four different field values, see Fig. 4b and 4c. At low temperature and zero magnetic field the system exhibit a superconducting state below a well-defined critical current of the islands $I_c^I = 2.5 \ \mu A$. For $I_{dc} > I_c^I$ the superconductivity is destroyed and a dissipative state emerges. The system obtain its normal resistance value above the critical current of the array $I_c^A = 20 \ \mu A$ (see Fig. D.1 in supplementary as it exceeds the axis limit in Fig. 4b). For B = 60 mT the superconductivity is lost and enters a normal state. Until this point the system retain a superconducting flavor in agreement with our interpretation of an intermediate phase where some superconductivity is still left. Increasing the temperature to $T = 800 \,\mathrm{mK}$ we clearly observe that crossing past the dashed line we enter a normal state where the current-voltage characteristic is completely flat (see Fig. 4c), and below this line dV/dI it remains a dip. To conclude, in the weak disordered limit we observe a superconducting (S) to normal (N) phase transiton with an intermediate state of still superconduting behavior.

In the more depleted limit (Fig. 4d) we notice a very different shape of the field-tuned transition as we increase the temperature. In this limit a reentrant superconducting state emerge at higher temperature. We observe two different kinds of transitions; a low temperature SI transition, when a peak in resistance at low I_{DC} is observed and at high temperature a SN transition, when the resistance is flat for all values of I_{DC} . We capture the SI transition at fixed temperature (100 mK) and measure the current-voltage characteristic for different field values (Fig. 4e). At low field we have a trace of superconducting behavior, showing dips in dV/dI. We see a finite resistance, but it is so low compared to the normal state resistance that it must be related to superconductivity. As we increase the field, the dip turns into a peak, indicating an insulating state. The dip-to-peak turnaround shown in Fig. 4e happens when the resistance of the sample is equal to $R_x \sim 13 \,\mathrm{k\Omega}$, ie. at the resistance value where we see the transition to the insulating state in Fig. 2. In reference to Fig. 4d, we have SI transition in the low temperature limit and a SN transition at high temperature. The separation line is defined from current-voltage measurements between the two temperatures presented here and we see the peak dissappear in dV/dI when the temperature is ~ 500 mK and remain flat for all temperatures above.

We end the current-voltage study in the very depleted

limit, where the system exhibit a complete isulating state at low temperature and for all field values, represented in a giant peak in dV/dI at low dc-current (Fig. 4h) and suggests a strongly blockaded system. Increasing the temperature in this regime surpresses the insulating state, shown in a split peak at low dc-current (Fig. 4i). The slight dip indicate a trace back to the orginal superconducting state, in the form of a reverse transition, however, the system remain highly resistive and an insulating state is still preserved.

We measured both samples A and B in the weak disorder, strong disorder and very strong disordered limit and observe same characteristic phase planes and transitions, see Fig. C.1 in supplementary.

Appearance of commensurability effects in JJA— In a Josephson junction array the periodic structure leads to strong commensurability effects. We observe an interplay between two different effects where the first effect is not related to the periodicity of the array. The effect appears at relative high fields where we observe strong temperature dependent waterfall-like features (horizontal thin stripes in Fig. 4a) appear, and is equally spaced ~ 4 mT after the first event at ~ 12 mT at low temperature (see supplemented Fig. 3.3).

This effect is related to the size of each individual square island and indicate the fields where one vortex enters the Al islands. The vortex penetraion field decrease uniformly with increasing temperature and the first strong feature where one flux penetrates is in agreement with ref. [11, 12], reporting on penetration fields as a function of temperature for a $1\mu m$ square Al island. Below the flux penetration field we have a complete superconducting state (*Meissner state*).

The second effect is related to the periodicity of the square array. Magnetic flux enter the array in quantized values so an integer number of vortices passes through each corner [13], hence impose phase frustration between the islands. The array reside in the superconducting state at integer values of frustration f (the average number of flux quanta per unit cell) and at any rational fractions, i.e. f = 1/2 where the array has been half filled with vortices. However, small changes can easily disrupt the commensurability and lead to dissipation when the vortices become mobile. We define the frustration parameter $f = B/B_0$, where $B_0 = \Phi_0/A$ with A beging the area of a unit cell $(A = (a + b)^2)$ and $\Phi_0 = h/2e$ is the flux quanta. We find $B_0 = 1.8 \text{ mT}$ in agreement with our observation of the first strong frustration minima (see supplemented Fig. 3.2).

Transition from frustration dip-to-peak

In a Josephson junction array the vortices organize in the lattice at every commensurated fields and reside in a superconducting state, as explained previously. The vortcies are freezed into a very defined arrangement as





Figure 5: Current-driven transition from frustration dip-to-peak a A color map of transition from frustration dip to frustration peak upon increasing the applied current I_{DC} . b Line cuts of the differential resistance show dips turning into peaks of frustration filling f = 1/2 and f = 1. c Frustration filling f = 1with the left and right seperatrix, used to perform scaling analysis, presented in d for the left branch, with a critical exponent $\epsilon_L = 1.5$ and e for the right branch with $\epsilon_L = 1.7$.

an effect of trapping at pinning sides of the array and due to mutal repulsion leading to a strong localizing action [14]. This state of localized vortices at commensurated fields, meaning an integer number of vortices per pinning, has previously been studied in terms of a vortex Mott insulator [15] due to a current-driven transition from a state of locked vortices to dynamic vortices at commensurated fields, shown in transition of frustration dip to frustration peak. The important observation in this study is the fact that dV/dI turns into a peak at each frustration filling while the resistance R = V/Iremain a dip. This is interpreted as a still pinned system even at high dc-current, since resistance would capture dissipation caused by depinning [15],

We here report on a current-driven transition from a frustration dip to frustration peak in the measured differential resistance dV/dI performed in a dc-current setup with an ac-excitation current of 5nA applied. We measured both samples A and B, and observe the same kind of transitions at commensurated fields when applying an electric current. Both samples were studied at different gate voltages. We here present data from device B in the limit where the gate is least negative. In this sample the superconducting islands are slightly further separated, which changes B_0 to 1.5 mT in the frustration parameter $f = B/B_0$. We observe pronounced frustration dips corresponding to f = 1/2 and f = 1 and moderate dips at rational fillings f = 1/4, 1/3 and 2/3, shown in Fig. 5a. Each of them turn into peaks when an electric current is applied, where strong signatures of both f = 1 and f = 1/2 lights up in a candle-like fire when approaching the critical current corresponding to each frustration feature (Fig. 5a). Integers are more robust as shown in Fig. 5a and 5b compared to fractional fwhere less modulation is required to disrupt the commensurability [13]. The dip-to-peak evolution of rational fillings f = 1/4, 1/3 and 2/3, show a less pronounced transition while f = 0 indicate a different behavior of two large peaks emerging on each side and remain two separate peaks until destroyed by the applied dc-current. The two separate peaks mirrored around f = 0 can be interpreted as a state of not exactly zero, but an equal mix of residual vortex and antivortex, where field flips one of a kind and a dip-to-peak transition at zero frustration is not achieved in this limit. In a limit where the fixed gate voltage is more negative, we drive a transition from dip-to-peak at f = 0 by applying a dc-current. In this limit we only see a vague signature of f = 1 and no frustration dip or peak at f = 1/2 (see supplemented Fig. D.2). The more neagtive gate voltage surpresses superconductivity, hence the commensurability.

Detailed plots of dV/dI as a function $f = B/B_0$ are shown in Fig. 5b, presenting frustration dips-to-peak in the differential resistance. Similiar to ref [15] we find that at each frustration filling the resistance R = V/I remain a dip (see supplemented Fig. 3.4).

Scaling model of f = 1 — We now characterize the transiton investigated in the critical region of f = 1, (see Fig. 5c), where scaling theory predicts that measurable quantities, in our case is dV/dI, follow scaling laws near the phase transition. We perform a scaling of the order parameter, the applied dc-current I, similar to [Valerii] of frustration f = 1. We define $|I - I_0| \propto |b|^{\varepsilon}$ at the critical point (I_0, f_c) , where f_c is the field frustration value, and I_0 is the current of the separatrix. We write the scaling relation of the form

$$\frac{dV(f,I)}{dI} - \left[\frac{dV(f,I)}{dI}\right]_{I=I_0} = \mathcal{F}\left(\frac{\mid I - I_0 \mid}{\mid b \mid^{\varepsilon}}\right). \quad (2)$$

We perform an independent scaling of both left and right side of the critical frustration. We note an asymmetry of left and right side observed for the integer frustration filling f = 1, and similarly in device A for f = 1,2 and 3, while the fractional frustration f = 1/2 remain symmetric around f_c . The asymmetry leads to different separatices found from the condition $d(dV/dI)/df |_{f=f_c} = 0$, which is where dV/dI is flat when approaching f_c and shown in Fig. 5c. When performing the scaling analysis we subtract dV/dI with the separatrices independently, where $I_0^L = 2.5\mu$ A for the left-hand side and $I_0^R = 2.35\mu$ A for the right-hand side. We find the expression for $b = f - f_c$, where $f_c = 1$ in this case and $f = B/B_0$. We see a slight change in the value of B_0 as we increase the electric current; the commensurated field value decrease with increasing current and will be taken into account in the scaling analysis, leading to $b = B/B_0(I) - 1$.

We find the best exponent by fitting a straight line to a plot of logarithmic values of $d(dV/dI - dV/dI(I = I_0^{L/R})/dI)$ and 1/b, revealing a critical exponent for f = 1 equal to $\varepsilon_L = 1.5$ for the left side and $\varepsilon_R = 1.7$ for the right side. Fig. 5d and 5e are scaling plots of left and right branch respectively, showing $dV/dI - dV/dI(I = I_0^{L/R})$ as a function of the variable $|I - I_0^{L/R}|/|b|^{1.5}$. The left branch give a better scaling result, with points collapsing together on the same line, while the right branch show more spread of the data.

Our study of a current-driven transition from frustra-



Figure 6: In-plane magnetic field effect on disorder-temperature driven SIT . a $B_{||} = 275$ mT. The measured resistance as a function of temperature. Each curve correspond to a different V_{TG} value. The separatrix is highlighted with a dashed line at $V_{TG} = -2.80$ V, and correspond to critical resistance value close to $R_x \sim R_Q$. b Commensuration effects in an in-plane field. At zero field the system shows

incommensurability. At $B_{||} = 150$ mT frustration has disappeared. **c** Measured resistance as a function of V_{TG} . All temperature curves cross in a single point at $R_x \sim R_Q$. **d** Scaling plot showing improved scaling and a critical exponent $\alpha = 0.1$ corresponding to $z\nu = 10$. **e** Collapse of critical exponents when increasing $B_{||}$. tion dip to peak, revealed scaling of the order parameter I for a collection of frustration filling values. The scaling analysis of integer filling f = 1 yielded a critical exponent of 1.5 for the left side and 1.7 for the right side. and fractional filling f = 1/2 gave a critical exponent of 2, which is different from previously reported studies, where a critical exponent of integer frustration values was found to be 2/3 and for frustration filling f = 1/2the critical exponent was equal to 1/2 [15]. We conclude that our semiconductor Josephson junction array can be used to study a vortex insulating state driven into a state of wandering vortices when applying an electric current. We find the same indications of a previously reported vortex Mott insulator to metal transition citeValerii, however, the difference in exponents lead to the open question that the transition of frozen vortices into a dynamic state may have different kinds of origins.

Collapse of critical exponent in an inplane magnetic field

We end this study by taking the advantage of a new knob to affect the supercondutivity of our 2D proximity-coupled array. We have already seen that a magnetic field applied perpendicular to our system has a detrimental effect on the superconductivity, leading to a state of normal resistance value when the gate is least negative and in a more depleted limit we drive the system through the SIT, hence obtain a field-tuned insulating state. In a thin film where thickness $d \ll \xi_0$ $(\xi_0^{Al} = 1.6 \,\mu\text{m} \, [16])$ we have the ability to suppress the superconductivity by the Zeeman energy in InAs. Previous studies of this material [17] found a critical in-plane field value of the thin Al film equal to $B_c = 1.65$ T.

We study the effect of an in-plane magnetic field in a gate-temperature driven phase transition. We present data on device B, which qualitatively behaves as device A, and study a gate-temperature driven SIT at different values of an applied in-plane magnetic field. We here show results at $B_{||} = 275$ mT as an example, see Fig. 6a. In reference to Fig. 6b, we show the effect of an applied in-plane field on commensuration effects. At zero field the system shows incommensurability, as in the old sample, as field is increased the commensurability becomes weaker until $B_{||} = 150$ mT, where the frustration has disappeared.

In Fig. 6a the measured R_s as a function of temperature show the transiton from superconduting state to an insulating state at $B_{||} = 275$ mT. The separatrix is found for the critical gate-voltage value $V_x = -2.80V$ (dashed line in Fig. 6a) where the resistance is constant in the temperature interval. Below the critical value we bring the system into an insulating regime and we measured the SIT up to seven orders of magnitude in R_s (Fig. 6a). At temperatures ~ 1.6 K we still capture the superconducting transition of the Al islands.

We compare the new study to the gate-temperature driven SIT studied at zero field. We remark two observations; first, we note the separatrix at $V_x = -2.80V$, correspond to a critical resistance value very close to the quantum pair resistance value $R_Q = 6.45 \text{ k}\Omega$. We plot R_s as a function of V_{TG} with curves representing different temeratures. The curves cross in a well defined point almost at R_Q , shown in Fig. 6c. We performed a scaling analysis according to equation (1) and found $\alpha = 0.10$, corresponding to $z\nu = 10$. In Fig. 6d we have plotted R_s scaled with respect to R_Q , and both branches meet in a single point at $R_s \simeq R_Q$.

The second observation is the disappearence of an intervening metallic state. The transition we observe with an in-plane field applied can therefore be regarded as a direct transition from a superconducting to insulating state. The scaling analysis revealed an exponent $\alpha = 0.10$. In addition to, the data collapse on both the upper and lower branch with almost no deviation, indicating that the scaling has improved significantly. We understand the improved scaling in the context of a direct SIT. The scaling relation presented in (1)assumes a continuous and *direct* SI transition [1], where our previous study presented in Fig. 1, show indication of an intermediate metallic state. With an in-plane field we have destroyed the intervening state, leading to a direct SIT and an improved scaling close to the phase transition. This also eliminates the possible heating effect as we cool down for being the source of the saturating curves in Fig. 1.

Values of α for different in-plane field values are presented in Fig. 6e. The critical exponents as a function of $B_{||}$ diverges, suggesting that α collapse as we approcah the paramagnetic normal state.

In conclusion, our superconductor/semiconductor Josephson Junction array have provided a very tunable system, enabling a continuous phase transition from a superconducting state into a insulating state. We performed a scaling analysis in the gate-temperature driven phase transition revealing a critical exponent comparable to quantum percolation. We further studied the JJA in a three parameter phase plane and was able to field-tune the system into a normal resistive state when the gate voltage is least negative, while close to the critical resistance value we were able to field-tune the system into an insulating phase at low T. We observed pronounced commensurablity effects related to the periodic structure of the array. Applying an electric current we drove the system from a frustration dip into a frustration peak, suggesting localized vortices at low current and a state of dynamic vortices at high current. We observed this type of transition in a wide collection of frustration fillings. Additionally the system respond to in an in-plane

and a direct gate-temperature driven supercondutor-toinsulator transition was presented with improved scaling and a critical exponent collapsing as a function of higher in-plane field values.

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Appendix B

Fabrication steps

B.1 Mesa patterning

The first step of the fabrication of the sample is mesa patternig. This step is where Hall bars and alignmentmarks are defined by e-beam lithography, then etched using a wet etch procedure. A 2.5×5 mm chip is first cleaved off the JS118 wafer where the optimum is to aim for good spots of the wafer, which means staying away from the edge where non-uniformities in the eptixial aluminum often occurs. These bad regions in the aluminum have been shown to cause a lot of troubles and inconsistent etching of the aluminum. Since the aluminum arrays are etched, choosing a good region from the beginning can be crucial for how well defined each island of the periodic array turns out to be.

The mesa patterning include e-beam lithography of the designed pattern and then etched in a three-step etching procedure, elaborated in the listed instructions below.

 \triangleright Clean and spin resist After cleaving the chip, rinse it for 2 min in Millipore (MP) water, 2 min in acetone, 1 min in IPA and blowdry with N₂.

Spin PMMA A4 using acceleration and time setting 4000 rpm, 45 seconds. Center the chip on the resist spinner, start the selected program and pause at 500 rpm while dispensing 2 drops of PMMA A4. Start program and it will spin up to the maximum acceleration of 4000 rpm and spin for 45 seconds. Bake 3 mins at 185 $^{\circ}$ C.

- ▷ E-beam Lithography In this step both mesas and alignmentmarks are patterned. The alignmentmarks will be used in the proceeding fabrication steps and it is therefore important that these are exposed with a low current setting whereas the large features will be written with a higher current setting.
 - **High definition** alignmentmarks : I = 500 pA, writing field = 300 μ m, 60.000 dots and dwell time of 0.4 μ S/dot. Resulting in a dose of 800 μ C/cm².
 - Low definition mesa: I = 20 nA, writing field = 600 μ m, 20.000 dots and dwell time of 0.36 μ S/dot. Resulting in a dose of 800 μ C/cm².

After finishing the exposure the chip is developed and plasma etched to remove leftover resist residues.

- 1. Develop 60 s swirl in MIBK: IPA in ration 1:3. Rinse in IPA and blowdry with N_2 .
- 2. Plasma etch 60 s, which removes ~ 15 nm of PMMA.
- ▷ Mesa etch The recipe for mesa etch is developed by the 2DEG team (Moretn Kjærgaard, Fabrizio Nichele and Henri Suominen) after struggling with many issues. For more details than provided here, I refer to ref. [1]. The etching of mesas is a 3 step cycle of first removing the Al top layer, then etch the mesa, where the depth is set to be sure that no parallel conduction paths between nearby mesas remain after etching. The last Al etch secure that not Al residues are left.

First prepare the two etchants:

- 1. Pour standard Aluminum Etchant Type D for Transene into a 50 mL plastic beaker and place it with another beaker of Millipore water in a 50°C hotbath. Keep a large beaker (~ 100 mL) filled with MP water next to the hotbath. It will take ~ 10 min for the etch to thermalize and should be checked with a thermometer before etching of Al.
- 2. In the meantime the mesa etch is prepared by mixing H_2O : citric acid : H_3PO_4 : H_2O_2 in the ratio 220 : 55 : 3 : 3. Since H_2O_2 is the oxidaizing agent it should be added in the end to obtain a consistent etching rate of ~ 0.5 nm/s. Start by mixing 220 mL H_2O in a beaker (750 mL) and place in with a magnet in the bottom of the beaker on a magnet stirrer.
- **3.** Pour in 55 mL citric acid (1M) and start the magnet stirrer.
- 4. Take out the H_2O_2 stored in a refrigerator as it needs to thermalize at room temperature before adding it to the etchant. Storing it in a cold place will keep its lifetime longer.
- 5. Add 3 mL of H_3PO_4 to the mixture and keep stirring.
- 6. Finally add the H_2O_2 after it has been ~ 5 min at room temperature.
- 7. Check the temperature of the Al etch with a thermometer and make sure the temperature is within $\pm 1.5^{\circ}$ as the rate depends on the temperature [1].

Now the etch cycle can begin:

- 8. Use an acid tweezer and swirl 10 s in the Al etch. Rinse 20 s in warm MP water and finally rinse 40 s in fresh MP water. Blowdry both chip and tweezer N₂.
- **9.** Etch for 10 min (600 s) in the mesa etch. Keep the magnet stirrer on to have continous motion of the etchant. The setting on the magnet stirrer should correspond to ~ 1 Hz. Rinse in fresh MP water for 40 s and finish with blowdry of N₂.
- 10. Finally etch one more time in the Al etch. 10 s in the Al etch and 20 s in warm MP, finish with a rinse in fresh MP and blowdry with N_2 .
- 11. Remove the resist mask using \sim 50°C acetone for 5 min and rinse in IPA for 1 min. Blowdry with N².

B.2 Island patterning

The island patterning is a delicate procedure, where doses for the e-beam lithography step has been optimized along with ething time and temperature of the Al etch. The Al has been found to depend a lot on the quality of the Al itself, and therefore it is highly recommended that the Al surface look uniform as it will cause problems when the island pattern needs to be done. Two scenarios of and bad etch can be seen in Fig.

▷ Clean and spin resist Rinse the chip for 2 min in MP water, 2 min in acetone, 1 min in IPA and blowdry with N2.

Spin PMMA A2 using acceleration and time setting 4000 rpm, 45 seconds. Center the chip on the resist spinner, start the selected program and pause at 500 rpm. Dispensing 2 drops of PMMA A2 and start program. Bake 3 mins at 185 °C.

- ▷ E-beam Lithography I = 100 pA, writing field = 150 µm (corresponding to the size of the array pattern), 60.000 dots and dwell time 0.38 µS/dot, leading to a dose of 600 µC/cm². After finishing the exposure the chip is developed and plasma etched to remove leftover resist residues.
 - **1.** Develop 60 s swirl in MIBK:IPA in ration 1:3. Rinse in IPA and blowdry with N^2 .
 - **2.** Plasma etch 60 s, which removes ~ 15 nm of PMMA.
- ▷ Aluminum etch The Al etch recipe has been developed after testing a bunch of different combinations of time and temperatures to prevent the etch from running too far under the PMMA where it starts etching the islands that I wish to remain as well defined as possible.

The best combination of time and temperature is 11 s at 46°C . Make sure the temperature is within $\pm 0.5^{\circ}$. Test with a thermometer before proceeding the etching of the island pattern. One way to check the etching if you don't want to waste a whole sample, is to make test windows in regions where it dose not affect the device. I placed three different array patterns around the alignmentmarks where aluminum was still left. The test patterns are then examined before removing the resist, using E-beam imaging. Then a second etch can be made if there is still Al left between the islands. In this way the etch is more controlled and underetching is more safe than overetch and detroy the sample.

- 1. Use an acid tweezer and swirl 11 s in the Al etch. Rinse 20 s in warm MP water and finally rinse 40 s in fresh MP water. Blowdry both chip and tweezer N₂.
- 2. Load the sample in the provided E-beam system (using 10kV acceleration voltage) where imaging is allowed. Search for your test windows and check your etch. If the etch is good and no Al is left between the islands proceed and move to step 3. otherwise jump back to 1.
- 3. Remove the resist mask using $\sim 50^{\circ}$ C acetone for 5 min and rinse in IPA for 1 min. Blowdry with N₂.



Figure B.1: Aluminum etch of two different samples. a Sample JS118.11 from a good area of the wafer, show no residue between the Al islands after etching, while b sample JS118.12 from a bad area of the wafer, show residue in between the islands.

B.3 Deposition of insulator

For deposition of the insulating layer between the Al pattern and the top gate I use a atomic layer deposition (Cambridge Nanotech Savannah ALD). I define a recipe in the program provided by the system with settings 400 pulses of trimethylaluminum (TMA), using H₂O as oxidizing agent. In then end this result in ~ 40 nm Al₂O₃ oxide layer. The steps are as follows:

- \triangleright Clean Thorough cleaning of the chip, using first a 3 min clean in 1,3-Dioxolane at ~ 50°, then 2 min in Acetone and a finishing step of 1 min in IPA. Blowdry with N₂.
- \triangleright ALD deposition Place the chip in the center of the ALD machine and close the lid. Start the program which usually takes ~ 17 hrs.

When the program has finished take out the chip of the ALD machine and store safely until deposition of the top gates.

B.4 Deposition of gates

The final step of fabrication of the SIT sample is deposition of metallic top gates. The gates are made in one final E-beam lithography step followed by metal evporation. The gates are of the size of serveral microns and there is no need to do a high definition lithography step here.

 \triangleright Clean and spin resist Rinse the chip for 2 min in MP water, 2 min in acetone, 1 min in IPA and blowdry with N₂.

Spin EL6 using acceleration and time setting 4000 rpm, 45 seconds. Center the chip on the resist spinner, start the selected program and pause at 500 rpm. Dispensing 2 drops of EL6 and start program. Bake 3 mins at 185 °C. Spin PMMA 4 using same procedure and setting as described above. Bake for 5 mins at 185 °C.

ightarrow **E-beam Lithography** I = 20 nA, writing field = 600 μ m, 20.000 dots and dwell time 0.36 μ S/dot, leading to a dose of 800 μ C/cm².

After finishing the exposure the chip is developed and plasma etched, using the standard recipe:

- **1.** Develop 60 s swirl in MIBK:IPA in ration 1:3. Rinse in IPA and blowdry with N^2 .
- **2.** Plasma etch 60 s, which removes ~ 15 nm of PMMA.
- ▷ Metal evaporation The evaporation is performed using AJA International ATC-E (E-beam evaporation). A layer of titanitum (Ti) followed by a layer of gold (Au) are evaporated:
 - 1. 10 nm Ti at an angle of 10° with rotation setting at 45 RPM
 - 2. 150 nm Au. The first 20 nm of Au has been evaporated at an angle of 10° and rotation setting at 45 RPM. The remaining 130 nm is evaporated with an angle of 0° and same rotation.
- ▷ Lift-off For the lift-off process, the chip is place in a beaker of 1,3-Dioxolane for ~ 4 hrs. Then heated in a hotbath to 50° for 10 min. Finish lift-off process with a needle attached to the end of the N₂ gun and carefully blow over the sample, while it is still in the beaker of 1,3-Dioxolane. This will help remove the flaps of metal hanging of the edge of the mesas. Use an optical microscope to see that only the top gates remain and finish lift-off process by cleaning the sample in IPA and blowdry with N₂ with no needle attached.

Appendix C

Device A and B: Phase planes



Figure C.1: Thermodynamic phase planes for the two devices A and B.

Appendix D

Supplemental figures



Figure D.1: Current-voltage measurement of the Josephson junction array. a Measured voltage as a function of applied current. The lower subpanel show the critical current of the array $I_c^A = 20\mu$ A and b the critical current for the island $I_c^I = 2.5\mu$ A.



Figure D.2: Zero field turnaround from minia to peak. In a more depleted limit with gate at $V_{TG} = -3.3$ V for device A. a,b The measured differential resistance, showin a dip-to-peak at zero field. c,d The measured resistance remain a dip for all values of applied current.