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Determination of Reference Velocities in the North Atlantic based on Physical Oceanographic Inverse Methods

[Danish: Bestemmelse af Reference Hastigheder i Nord Atlanten baseret
på Fysiske Oceanografiske Inverse Metoder]

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Abstract

Anomalous ocean currents are linked to melting ice caps [Cuffey & Paterson 2010][1] and global changes in the environmental conditions [C. Wunsch 1996][2]. These changes may severely impact life on Earth, as has been discussed extensively, but inconclusively, in the ongoing climate debate. To understand and forecast the environmental changes, accurate quantification of geostrophic velocity profiles is of profound importance [C. Wunsch 1996][2]. Ocean currents at the surface are being measured directly and continuously by satellite tracking of free drifting probes deployed by the Global Drifter Program (GDP) administered by The National Oceanic and Atmospheric Administration (NOAA). However, the only convenient means for quantifying deep sea currents is to estimate these based on deep sea measurements of salinity and temperature at different depths and hydrographic positions. Deep sea measurements are time consuming and require a manned vessel to be on the measurement site, thus making each measurement costly and limiting the feasible number of measurement sites. Several mathematical methods for converting a set of measurements into geostrophic reference velocities have been proposed [Pond & Pickard 1978][3], including the box model method [C. Wunsch 1996][2] and the level of no motion method. However, it remains unclear which method perform best given a sparse set of measurements. In this manuscript I present a quantitative comparison of the box model method and the level of no motion method. This comparison takes departure in a sparse set of 4 deep sea measurements obtained during a cruise from Denmark to Iceland (Aug. 27. - Sep. 2., 2012) on board research vessel Dana (R/V Dana). To quantify the performance of the two methods I compare the resulting geostrophic reference velocity estimates to measurements from the GDP drifter Data Assembly Center (DAC)[Lumpkin & Johnson 2013][4]. Based on the four measurements, I show that the level of no motion method deviates by $[0.3 \sigma ; 1.6 \sigma]$, to the reference velocities from GDP, while estimates from the box model method deviates much more significantly by $[2.5 \sigma ; 7.5 \sigma]$. Hence, my observations suggest that the level of no motion is the preferable tool for estimating geostrophic reference velocities from a sparse set of measurements. Based on these finding I suggest further investigations in which measurements from an Acoustic Doppler Current Profiler (ADCP) are available to compare with estimated reference velocities.

Danish: Resumé

Ab normale havstrømme korrelerer med smeltende iskaper [Cuffey & Paterson 2010][1] og globale klima- og miljøforandringer [C. Wunsch 1996][2]. Disse forandringer påvirker livet på jorden og har været grundlag for, intense men konklusionsløse, diskussioner i den igangværende klimadebat. For at forstå og forudsige de miljømæssige forandringer, er præcise beregninger af den geostrofe hastighed af afgørende betydning [C. Wunsch 1996][2]. Havoverfladestrømme er blevet målt kontinuerligt af satellitsporede drivende bøjler, som er søsat under The Global Drifter Program (GDP) - administreret af The National Oceanic and Atmospheric Administration (NOAA). For at estimere dybe havstrømme er salinitet, temperaturmålinger fra forskellige dybder og hydrografiske positioner nødvendige. Dybhavsmålinger er tidskrævende og kræver et bemandedt forskningsskib. Matematiske metoder er indført [Pond & Pickard 1978][3], til beregning af geostrofe referencehastigheder fra temperatur-og salinitets målinger. Følgende invers metoder, box model metoden [C. Wunsch 1996][2] og level of no motion metoden, vil blive benyttet i en undersøgelse af, hvilken metode, der giver det bedste resultat udfra et begrænset sæt af målinger. I dette manuskript, vil jeg præcentere en kvantitativ sammenligning af box modellen og level of no motion modellen. Min Sammenligning tager udgangspunkt i et begrænset antal målinger, foretaget under et togt fra Danmark til Island (Aug.-27. - Sept. -2. 2012) om bord på research vessel Dana (R/V Dana). For at kvantifisere de to metoders nøjagtighed, sammenligner jeg de beregnede estimater af den geostrofe referencehastighed, med målinger fra GDP Data Assembly Center (DAC) [Lumpkin & Johnson 2013][4]. Ud fra de 4 havmålinger afviger level of no motion metoden $[0.3 \sigma ; 1.6 \sigma]$ fra referencehastigheden taget fra GDP, og box modellen afviger mere signifikant $[2.5 \sigma ; 7.5 \sigma]$. Ud fra disse resultater konkluderer jeg, at ud fra et begrænset antal målinger, er level of no motion metoden at foretrække til beregning af geostrofe referencehastigheder. På baggrund af disse resultater, ville det være at foretrække til fremtidige undersøgelser, at benytte hastighedsmålinger fra Acoustic Doppler Current Profiler (ADCP) til at sammenligne de estimerede referencehastigheder med.

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2 Introduction

Determination of ocean currents is of high importance in the investigation of climate changes. Investigations concerning which method is the most reliable, have occupied oceanographers for decades. A problem of how to compute the geostrophic flow of water from observations made from ships is often referred to as the classical problem of physical oceanography [C. Wunsch 1996][2]. The thermal relations is used to reduce the problem to the missing reference velocities.

In an investigation of how to compute the geostrophic velocity, observations was carried out on board R/V DANA during a cruise from Hirtshals, Denmark to Reykjavik, Iceland from August 27th to September 2nd 2012. The cruise was provided by the Technical University of Denmark (DTU) with the intention of giving the students a more practical knowledge of the disciplines of biological, optical, chemical and physical oceanography.

During the cruise 4 Conductivity-Temperature-Depth (CTD) casts were made during the cruise using a Sea-Bird SBE 9. The casts were made in the most northern part of the Iceland Basin (IB) and the Scotland-Shetland channel, just south of the Greenland-Scotland Ridge (GSR). The area's distinct characterization is the cold saline overflow water from the Danish Strait (DS) and the Iceland-Scotland Ridge. The overflow water originates from the Nordic Seas, where deep water formation occurs by cooling down the warm surface water from the south, which is transported by the North Atlantic Current (NAC). The cold, saline overflow water rises over the ridge and mixes with the warmer intermediate water. It is an area of interest because of the large amount of water transported from the surface to the ocean interior by the deep water formation.

In this paper, relative and absolute geostrophic velocity profiles will be derived from dynamic oceanographic methods [Pond & Pickard 1978][3]. Geostrophic transports are calculated to evaluate the sparse set of observations. Furthermore, a determination of a surface reference velocity is made using oceanographic inverse methods; the box model and the level of no motion method. In the level of no motion method a reference level is assumed at 600 m and 1000 m. Finally a comparison of the estimated reference velocities will be made to quantify which methods is preferable when only a sparse set of observations are available.

3 Theoretical background

Geopotential

To understand geostrophic flow the geopotential Φ has to be introduced. The geopotential is the work done to a mass M to rise it a vertical distance dz against the force of gravity [Pond & Pickard 1978][3]

$$\Phi = \int_0^z g dz \quad (1)$$

The geopotential is the gravitational potential per unit mass.[John M. Wallace & Peter V. Hobbs 2006][5]. In the atmosphere the geopotential changes with pressure. In the ocean the geopotential is a function of pressure but also a function of salinity and temperature, for that reason the geopotential depends on the density. Oceanographers use the convention of a specific volume $\alpha = \frac{1}{\rho}$. Now the geopotential distance between depths z_1 and z_2 can be formulated.

$$\Phi_2 - \Phi_1 = g(z_2 - z_1) = - \int_{p_1}^{p_2} \alpha_{35,0,p} dp - \int_{p_1}^{p_2} \delta dp \quad (2)$$

Where $\alpha_{35,0,p}$ is the specific volume of seawater with salinity 35 psu (practical salinity unit), temperature $0^\circ C$ and pressure p . δ is the specific volume anomaly. The first term to the right is called the standard geopotential distance $\Delta\Phi_s$, which is only a function of p , and the second part is called the geopotential anomaly $\Delta\Phi$, which is a function of p and ρ [Pond & Pickard 1978][3]. The geopotential anomaly is used in the thermal wind equation to calculate the relative velocity \mathbf{V}_{rel} .

Equations of motion

In general there are four forces that balance the acceleration term of a fluid, the pressure gradient force, the Coriolis force, the gravity and the frictional force. Forces in fluid dynamic are often referred to as force divided by mass, which is due to acceleration. One has to multiply with a given mass to obtain the force from the following equations. If one defines a coordinate system where the gravitation is along the vertical direction. The equations of motion are given by. [John A. Knaus 1997][6],

$$\begin{aligned}\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} &= -\frac{1}{\rho}\frac{dp}{dx} + fv + \chi \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} &= -\frac{1}{\rho}\frac{dp}{dy} - fu + \gamma\end{aligned}\quad (3)$$

Where u , v and w is the velocity components in x , y and z direction in Cartesian coordinates in the North/South-East/West coordinate system. ρ is the density of water, p is the pressure, f is the Coriolis parameter,

$$f = 2\Omega\sin(\phi) \quad (4)$$

depending of the latitude ϕ and the angular speed of the Earth rotation $\Omega = 7.292 \cdot 10^{-5} \frac{rad}{s}$ [Wallace & Hobbs 2006][5], χ and γ are frictional terms. In general $\frac{D}{Dt} \equiv \frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$ is the total derivative [John A. Knaus][6]. The total derivative describes the total change in velocity of a fluid particle, which is a sum of the change in its velocity over time plus the change in its velocity over distance. One can see from Eq. 3, that it is possible for a fluid to accelerate without a change in velocity over time. The first part on the right side of Eq. 3 is the pressure gradient force, second part is the Coriolis force, and third part is the frictional force. The equations of motion can be reduced and thereby contain less terms, depending on the oceanographic process. In this project, large scale ocean circulation is considered, and one has to figure out which forces that are dominating.

First we consider a fluid particle set in horizontal motion on a surface of equal potential. In this case there are no pressure gradient force and no frictional forces [John A. Knaus 1997][6]. The equations of motion can then be reduced to

$$\begin{aligned}\frac{Du}{Dt} &= fv \\ \frac{Dv}{Dt} &= -fu\end{aligned}\quad (5)$$

Here the acceleration is normal to the direction of flow and proportional to the flow velocity. This is an equation of a circle, where the fluid particle is accelerated towards the centre of the circle. The two forces, which balance each other, are the Coriolis force and the centrifugal force. The particle moves with constant speed $V = \sqrt{u^2 + v^2}$ along the circle path, this is called the inertial circle.

$$\frac{V^2}{r} = Vf \quad (6)$$

Where r is the distance to the centre of the circular path.

In small scale fluid dynamics the Coriolis force is generally neglected because of much stronger friction terms in the equations of motion. In the large ocean scale, the friction terms become smaller compared to the Coriolis term, so the Coriolis force have to be taking into account.

A method which oceanographers uses to determine whether the Coriolis force can be neglected or not is to consider the ratio of the centrifugal force to the Coriolis force in a given situation. This method results in a number, the Rossby number R_0 ,

$$R_0 = \frac{V^2/r}{Vf} = \frac{V}{fr} \quad (7)$$

Where R_0 is 1 for the inertial circle. R_0 is a measure of the validity of the geostrophic approximation [James R. Holton 2004][8], where $R_0 < 1$, results in the geostrophic approximation. A curvature with large radius and a fluid with relatively small velocity will give a small R_0 and the Coriolis force can not be neglected [John A. Knaus 1997][6] due to the geostrophic approximation.

Geostrophic flow

Assuming that the ocean currents are horizontal, the water will move along a surface of equal potential. The friction becomes sufficient due to no mixing. We have no gravitation term in the horizontal xy plane, which means that balance between the two forces, the pressure gradient force and the Coriolis force, occurs. This balance is called the geostrophic balance. One has to take into account that the pressure gradient is pointing in the direction of largest pressure and the pressure gradient force is pointing the opposite way of the pressure gradient. The geostrophic balance in the horizontal plane is given by [John A. Knaus 1998][6].

$$\begin{aligned}fv &= \frac{1}{\rho} \frac{\partial p}{\partial x} \\fu &= -\frac{1}{\rho} \frac{\partial p}{\partial y}\end{aligned}\tag{8}$$

Where u and v are geostrophic velocity components in respectively the x and y direction. Currents that obey this equation is called geostrophic currents. All major ocean circulations are approximately in geostrophic balance. [John A. Knaus 1998][6]

The direction of the geostrophic flow in the northern hemisphere is clockwise around a high pressure cell and counter clockwise around a low pressure cell. It is opposite in the southern hemisphere.

Ocean currents are made by the inclination between surfaces of constant pressure and surfaces of constant geopotential. To understand the process of oceanic currents physically, one has to consider a tank of cold and warm water. The cold water sinks and underlies the warm water because of the hydrostatic balance,

$$\frac{dp}{dz} = -\rho g\tag{9}$$

where g is the standard acceleration due to free fall $g = 9,80 \text{ m/s}^2$, which attends to keep the water at rest. When the cold water underlies the warm, it causes the warm water to rise and an inclination of the surface occurs. A horizontal pressure gradient occurs and a Coriolis force is therefore needed to balance the pressure gradient force. The water does not move down the inclination as a rolling ball on a hill. When the water begins to move downward, it turns to the right in the northern hemisphere and to the left in the southern hemisphere. This difference is due to the sign of the Coriolis force caused by the sine in its mathematical formulation, Eq. 4. A good rule of thumb is “light on the right” in the northern hemisphere.

Thermal wind equation

The thermal wind is not a physical wind, but a shear in the geostrophic wind as function of the height or depth. The thermal wind is caused by a horizontal pressure gradient, which occurs when cold, dense water meets warm, light water.

The thermal wind equations can be derived from the geostrophic balance, Eq. 8. [Pond & Pickard 1978][3] By multiplying both sides with ρ and differentiating the geostrophic balance with respect to z gives.

$$\begin{aligned}\frac{\partial(v \cdot f \cdot \rho)}{\partial z} &= \frac{\partial}{\partial z} \frac{\partial p}{\partial x} \\ \frac{\partial(u \cdot f \cdot \rho)}{\partial z} &= -\frac{\partial}{\partial z} \frac{\partial p}{\partial y}\end{aligned}\tag{10}$$

Changing order of the derivatives and using the hydrostatic balance, Eq. 9, gives.

$$\begin{aligned}f\rho \frac{\partial(v \cdot f \cdot \rho)}{\partial z} &= -g \frac{\partial \rho}{\partial x} \\ f\rho \frac{\partial(u \cdot f \cdot \rho)}{\partial z} &= g \frac{\partial \rho}{\partial y}\end{aligned}\tag{11}$$

Using the Boussinesq approximation, which states that density differences can be neglected except where they are multiplied by g [C. Wunsch 1996][2], gives the thermal wind equation.

$$\begin{aligned}f\rho \frac{\partial v}{\partial z} &= -g \frac{\partial \rho}{\partial x} \\ f\rho \frac{\partial u}{\partial z} &= g \frac{\partial \rho}{\partial y}\end{aligned}\tag{12}$$

The geostrophic calculations gives the relative velocity $\mathbf{V}_{rel} = \mathbf{V}_1 - \mathbf{V}_2$ which is a vectorial difference of two geostrophic velocities, where \mathbf{V}_2 is defined at a deeper depth, than \mathbf{V}_1 [James R. Holton 2004][7].

The thermal wind equation describes how fluids change their geostrophic velocities with depth, such fluids are called baroclinic fluids. In a baroclinic fluid, lines of constant pressure crosses lines of constant density. Fluids where lines of constant pressure are parallel to lines of constant density are called barotropic fluids. In a barotropic fluid there is no geostrophic shear with depth [John A. Knaus 1997][6]. The absolute geostrophic velocity is the sum of the baroclinic component and the barotropic component. To compute the absolute geostrophic velocity one have to know an absolute geostrophic reference velocity at some depth or the inclination of the sea surface relative to a gravitational equipotential [C. Wunsch 1996][2].

Geostrophic transport

Water with velocity \mathbf{V} , which flows through a cross area \mathbf{A} is called the volumetric flow rate, Q , and is given by.

$$Q = \mathbf{V} \cdot \mathbf{A} \quad (13)$$

In this project, volumetric flow rate will be referred to as the volume transport or transport. Normally the velocity changes with depth, and the total transport becomes the sum of the transport elements. If the depth interval becomes small enough, the sum becomes an integral.

$$Q = \int \int_A \mathbf{V} \cdot d\mathbf{A} \quad (14)$$

In the ocean, we assume, that the total transport through a volume defined by hydrographic stations is zero. Practically this would require a large number of observations. Furthermore it would require knowledge about the transport from both geostrophic and ageostrophic processes in the whole water column. This paper only concerns geostrophic transport. The total geostrophic transport can be used to evaluate of the model used. The unit of geostrophic transport is measured in Sverdrup (Sv) by convention. The relation between Sv and cubic metre per second (m^3/s) is,

$$1 \text{ Sv} = 10^6 \text{ m}^3/s \quad (15)$$

The classical problem of the geostrophic flow

One has to remember, that assuming geostrophic balance in an ocean is an approximation due to the reduced equations of motion, used in large scale fluid dynamics. The geostrophic flow loses fluid to a non-geostrophic flow near the boundary, so if one wants to calculate the real flow of the ocean interior, it is essential to know the ageostrophic components. Many oceanographers describe this as the classical problem. The question is, how to compute the geostrophic flow in deep oceans from observations made from ships [C. Wunsch 1996][2]. Practically one uses the thermal wind equation and the major issue is reduced to the missing integration constants, such integration constants are often called reference velocities. The reference velocity represents an absolute velocity at a reference depth z_0 , e.g. a surface velocity obtained from an Acoustic Doppler Current Profiler (ADCP), which is a hydro acoustic meter that measures water current velocities over a depth range using the Doppler effect¹ of sound waves, scattered back from particles within the water column, often mounted beneath the ship.

If the shape of the ocean surface were known relative to the gravitational equipotential, the flow field could be computed at all depths from the geostrophic relations. But how does one measure the surface elevation? Nowadays, the elevation by satellite altimetry could be measured.

Another way to estimate the reference level velocity is to measure a velocity at a reference depth. This method is inconvenient, because of daily and seasonally fluctuations. One has to average velocity measurements over longer periods to obtain a stable mean flow. Most oceanographic observations at sea are made by ships sailing with other purposes and it is not possible to tie up such ships for weeks or months [C. Wunsch 1996][2].

The problem of how to estimate a reference velocity has been discussed by oceanographers for decades. One method developed is called “level of no motion”, a method which is based on the assumption of an existing depth

¹A Doppler shift or Doppler effect, named after the Austrian Physicist Christian Doppler, is a consequence of a moving object sending out waves while moving. Because of the velocity of the object, a shift in frequency occurs corresponding to the direction of the movement relative to the observer

of no motion in the oceanic interior. The method will be further described later on. Subsequently Wunsch developed a method called the box model, which is based on the conservation of mass in an enclosed volume defined by hydrographic stations. These two methods are often referred to as inverse methods. In this paper a theoretical reference velocity will be determined by those two inverse methods for the purposes of evaluating the methods by a comparison to a mean reference velocity from a reference data set adopted from GDP [Lumpkin & Johnson 2013][4].

4 Experiment

Deep sea measurement procedure

This paper concerns the ocean physics of the North Atlantic Ocean, more specifically, a transect from Scotland to Iceland, where four CTD casts were made. A CTD is an instrument for oceanography, which measures the conductivity, Temperature and depth. The conductivity of the water is a measure of the salinity, the depth is measured in pressure of decibar and the temperature is measured in degrees Celsius.

The practical details and measurement procedure were discussed on late night lectures during the fare to the first station, and students were arranged in four groups, each group responsible for data in one of the four earlier mentioned disciplines. Students got access to control the CTD casts, under the teacher's supervision. In every station one shallow cast around 100 m and one deep cast around 1100 m was made. On the way down on every cast, measurements were followed on a monitor and the most interesting depths were decided in a discussion between students and teachers. Ten bottles were filled on the way up and each bottle contained five liters of water. Before the casts were taken, one student was in charge of distributing the water between the optical, chemical and the biological groups after their specific needs. A CTD sampler provides both bottle samples and continuous measurements during the cast. In this project the bottle data was not used, only the cnv. files, which is the format of the achieved data files produced by the Sea-bird (the CTD used). Two temperature and salinity sensors were mounted to the CTD sampler. In this project the average value of the temperature and salinity measurements were used. The time spent on each station was around one hour, the ship sailed on as soon as the CTD landed on board. Laboratory work and data analysis tasks were assigned and executed, while the ship was sailing to the next station. Date and time of each station are shown in table 1.

	Stations	Date	Time
Station 1	29/8 2012	13:39:51 pm	
Station 2	30/8 2012	09:59:27 am	
Station 3	31/8 2012	06:07:57 am	
Station 4	01/9 2012	05:35:11 am	

Table 1: Date and time of stations

Measurement locations

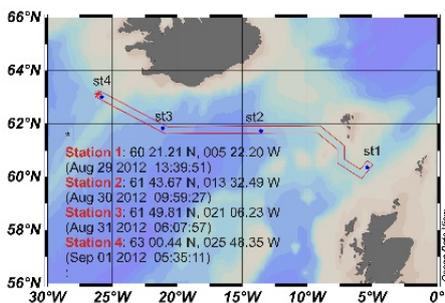


Figure 1: Map of stations taken on RV/DANA, coordinates are included.

Data from the four hydrographic stations were taken just south of the Greenland-Scotland Ridge (GSR), in the most northern part of the Iceland Basin (IB) and the Scotland-Shetland Channel (SSC). The area's distinct characterization is the cold saline overflow water from the Danish Strait (DS) and the Iceland-Scotland Ridge.

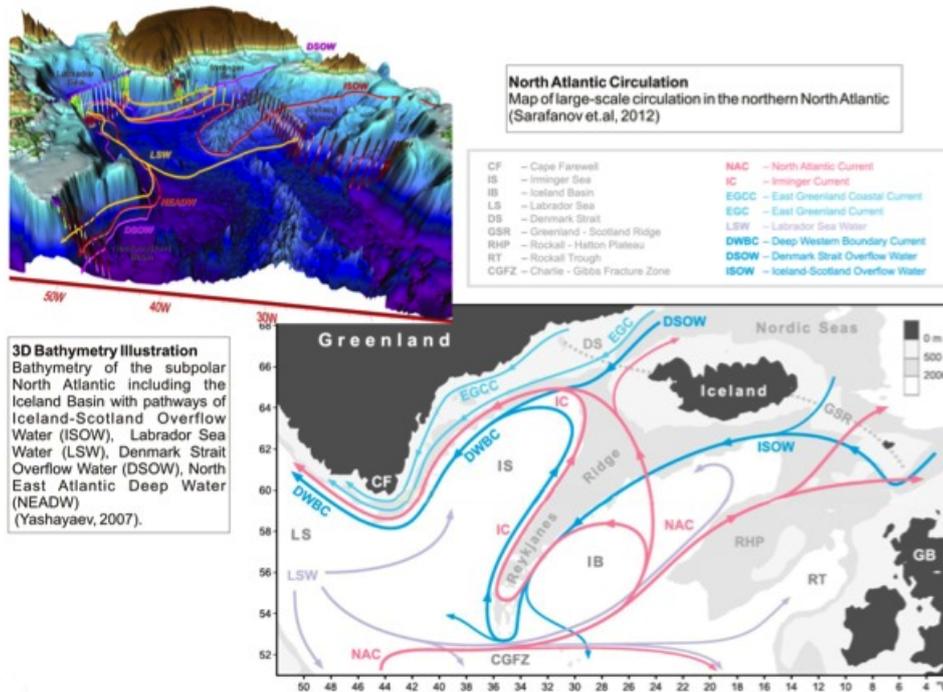


Figure 2: The upper part of the figure shows a map of the bathymetry of the North Atlantic Ocean and the lower part shows a map of characteristic ocean currents in the North Atlantic Ocean, where blue lines indicate dense boundary currents and red lines indicate light surface currents.

On figure 2, one can see the characteristic ocean current in the North Atlantic Ocean. The major NAC originates from the Gulf Stream and transports warm surface water northward to the Nordic Seas. The cold, dense southward overflow water constitutes of Iceland-Scotland Overflow Water (ISOW) and Danish Strait Overflow Water (DSOW), which originates from the Nordic Seas.

Reference velocity data set

In this paper a reference velocity \mathbf{V}_{data} from DAC is used to compute an absolute velocity \mathbf{V}_{abs} in a given depth. \mathbf{V}_{data} is a monthly mean surface velocity collected by drifters in an array of 1250 drifters. The positions of the drifters are interpolated and obtain a resolution of one degree latitude by one degree longitude. Every drifter is fitted with a thermometer, and measurements of the temperature in the atmospheric boundary layer are accessible.

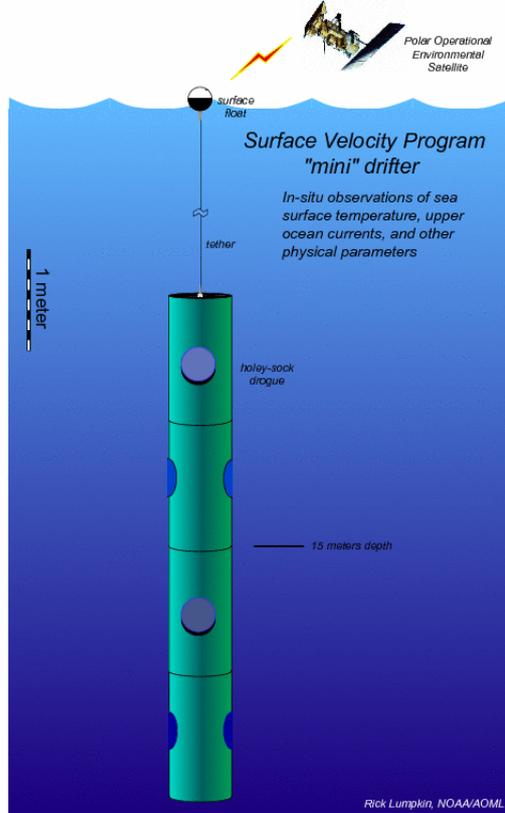


Figure 3: Drifter used to obtain the data from DAC. The drifter constitutes of a floating buoy and a drogue centered 15 m subsurface. Figure Rick Lumpkin, NOAA

The drifter consists of a floating buoy, which has a diameter of 30-40 cm and a drogue centred 15 m subsurface. The drogue catches the currents and pulls the buoy downstream. Furthermore, the drogue protects the drifter from strong wind and thereby avoiding slippery of the buoy. The drogue consists of multiple sections separated by rigid rings, which are conserving the cylindrical shape of the drogue. Each section has two holes rotated 90° relative to the next section to prevent the drogue from getting entangled. The drifter is positioned by a Doppler shift in the transmission frequency. The velocities are calculated from the differences in position. The drifters do not perfectly follow the water column averaged over the drogue depth of 15 m. For example, water are able to down well while the drifter is forced to stay at the surface. The resultant velocity measured is a combination of the flow of the water column, plus the upper ocean wind driven flow and plus the slip due to strong winds and waves. To minimize the slip, the tension between the buoy and the drogue has to be reduced to avoid noise, caused by surface wave motion, and the drag ratio has to be large. All measured data is sent to the satellites by a transmitter, which is mounted to the drifter [Lumpkin and Pazos 2006][9].

Velocities were low pass filtered with a cut-off at the 5 day period to remove high frequency variability in the drifters velocities, due to inertial oscillations, see Eq. 6, tidal effects and diurnal cycles. This results in an annual standard error on \mathbf{V}_{data} .

The data set is extracted in a total velocity field and a geostrophic velocity field. In this manuscript, only the geostrophic velocity field will be used. To compute geostrophic velocity, satellites are used. Satellites measure the sea surface height (SSH) of a drifter, but to compute the dynamic height, more accurate measurements are needed. Along-track sea level anomalies (SLA) relative to a seven years (1993-1999) mean profile is computed to derive the geostrophic velocity anomaly along the drifters trajectories [Hernandez & Rio 2003][10]. Using this method is only possible at locations more than three latitudes away from the Equator due to the

geostrophic relations. From the expression of the Coriolis force, it is seen that the sine causes the Coriolis force to be zero at the Equator. To obtain absolute geostrophic velocities, a mean dynamic velocity is added. The mean dynamic velocity is calculated by subtracting the velocity field, based on satellite altimetry, to the in situ measurement made by the drifter.

5 Results

Deriving of Relative and absolute velocity profiles from the geostrophic method

To derive the relative velocity \mathbf{V}_{rel} profile, calculations of the density profiles were made. In this paper the density has been calculated by using an oceanographic subroutine [McDougall & Barker 2011][11], which uses the Teos² 10 equations. The density distribution from each station was vertically integrated, with the surface as reference, to obtain the geopotential anomaly $\Delta\Phi$.

Furthermore, the thermal wind equation was used [Pond & Pickard 1978][3] to find \mathbf{V}_{rel} between two stations e.g. station 1 and 2 with $\Delta\Phi_1$ and $\Delta\Phi_2$. The direction of the Ship is assumed to be in the x' direction; as a result, I only use the x' equation of the thermal wind. This gives a thermal wind component v'_{rel} in the y' direction, where x' and y' is in a coordinate-system defined by the ship and the relative velocity. Velocities in the $x'y'$ coordinate system are denoted v' in the y' direction. Further, the notation $dx' = L$ is inferred, where L is the distance between two hydrographic stations in the $x'y'$ coordinate-system, e.g. 1 and 2.

$$v'_{rel} = v'_{ref} - v'_{abs} = \frac{1}{f} \frac{\Delta\Phi_2 - \Delta\Phi_1}{L} \quad (16)$$

Where v'_{ref} is a given reference velocity in the y' direction. The following figures show the salinity and temperature profiles of each station. Furthermore, the derived relative velocity is shown as a result of the salinity and the temperature profiles. The absolute velocity v'_{abs} is calculated using a projection of a known reference velocity \mathbf{V}_{data} from the data set [Lumpkin & Johnson 2013][4] onto the v'_{rel} .

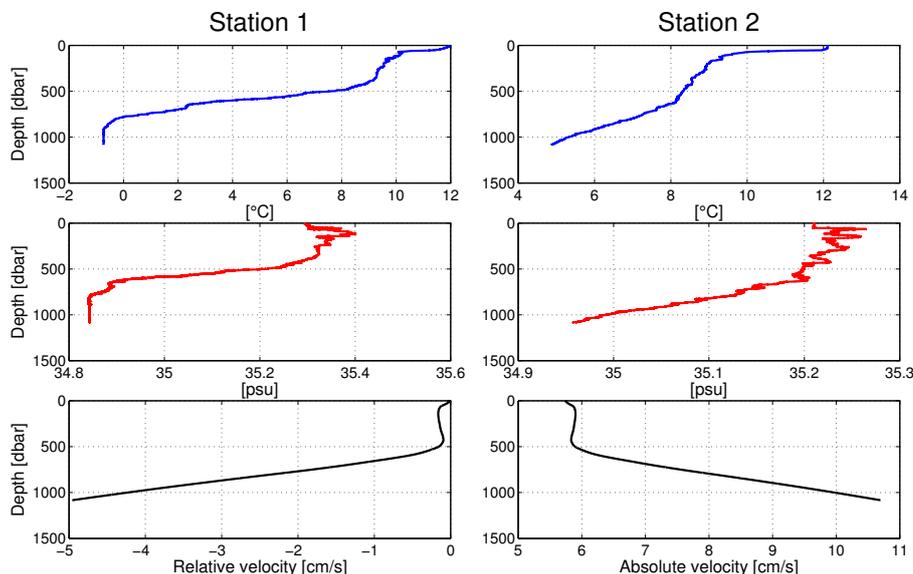


Figure 4: In the upper part, the figure shows the temperature and salinity profiles for station 1 and 2. In the bottom v'_{rel12} and v'_{abs12} is shown for both stations. A positive relative geostrophic velocity is directed to the left of the sailing path and a negative is directed to the right. Both directions are perpendicular to the sailing path.

²Thermodynamic Equation Of Seawater 2010

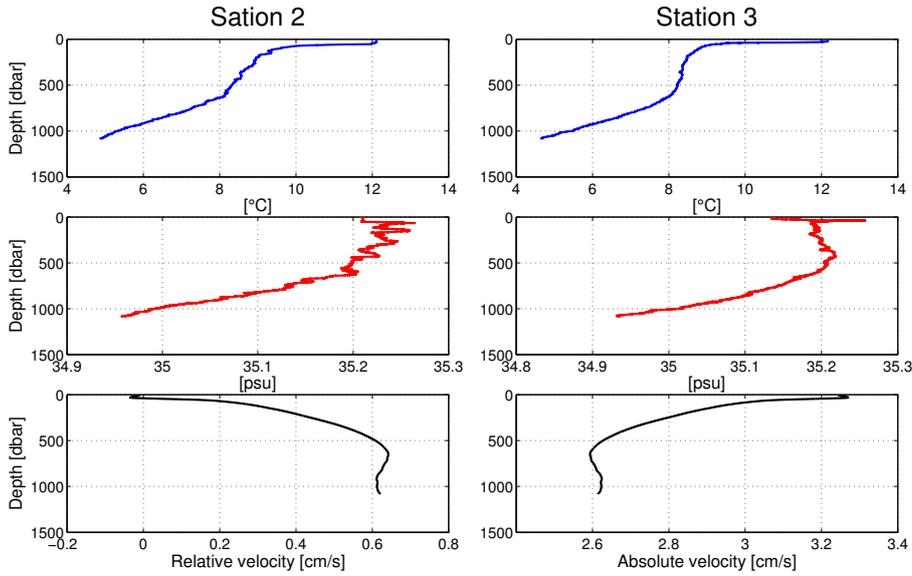


Figure 5: The figure shows the temperature and salinity profiles and the corresponding v'_{rel23} and v'_{abs23} profiles for station 2 and 3.

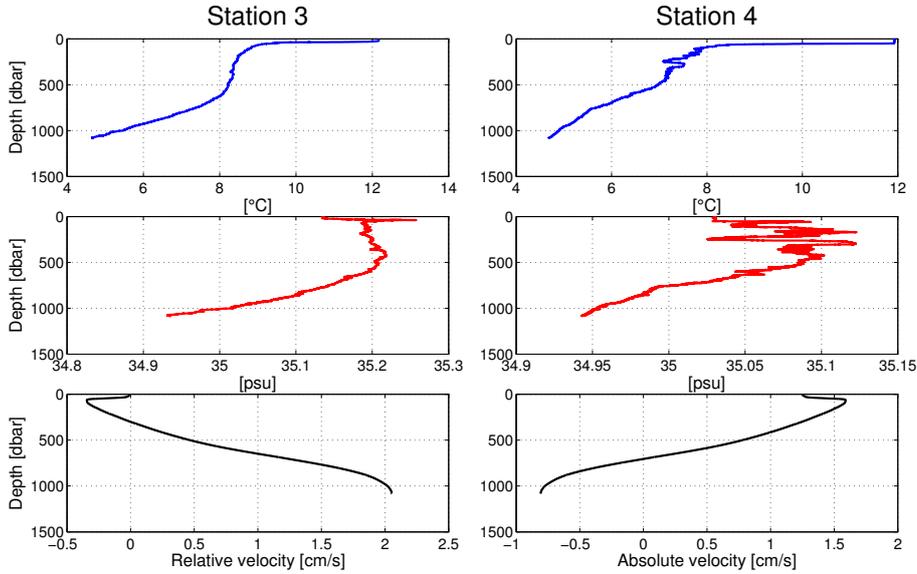


Figure 6: The figure shows the temperature and salinity profiles and the corresponding v'_{rel34} and v'_{abs34} profiles for station 3 and 4.

The remaining hydrographic station pairs, station pair 1-3 and 2-4, can be found on appendix A. All reference velocities were picked out of a velocity field from August by interpolation a of one degree latitude by one degree longitude data grid. As seen in table 1, station 4 is made early in the morning on the first of September, and a reference velocity from August velocity field is used. The explanation to this is based on the assumption that the actual surface velocity at 5.30 in the morning on the first of September does not vary much from the mean surface velocity of August. Velocity components between two stations were found by averaging the velocity components along the sailing path.

$$\begin{aligned}\bar{u}_{data} &= \frac{1}{N} \sum_{i=1}^N u_{datai} \\ \bar{v}_{data} &= \frac{1}{N} \sum_{i=1}^N v_{datai}\end{aligned}\tag{17}$$

Where i is the index number counting from 1 to N . For each 10^{-4} degree, a velocity is interpolated, and

therefore N is proportional to the distance between two stations. The \bar{u}_{data} and \bar{v}_{data} components were used to find a magnitude of the averaged reference velocity \bar{V}_{data}

$$|\bar{V}_{data}| = \sqrt{\bar{u}_{data}^2 + \bar{v}_{data}^2} \quad (18)$$

The following figures show the velocity fields from the data set [Lumpkin & Johnson 2013][4]. The positions of the hydrographic stations are included. The sailing path is assumed to be a straight line between each station pair. Fig. 7 and Fig. 8, respectively show the u_{data} and the v_{data} components fields. Fig. 9 shows the field of the magnitudes $|V_{data}|$. All figures show measurement fields from August in a Pcolor (Pseudocolor) plot. Pcolor plot is used instead of a surface plot. Pcolor plot makes one colour of each cell by bilinear interpolation. Each color represents an interpolated value of the velocity field. The interpolated colors are arranged in a color bar of minimum and maximum color, which are defined by the user.

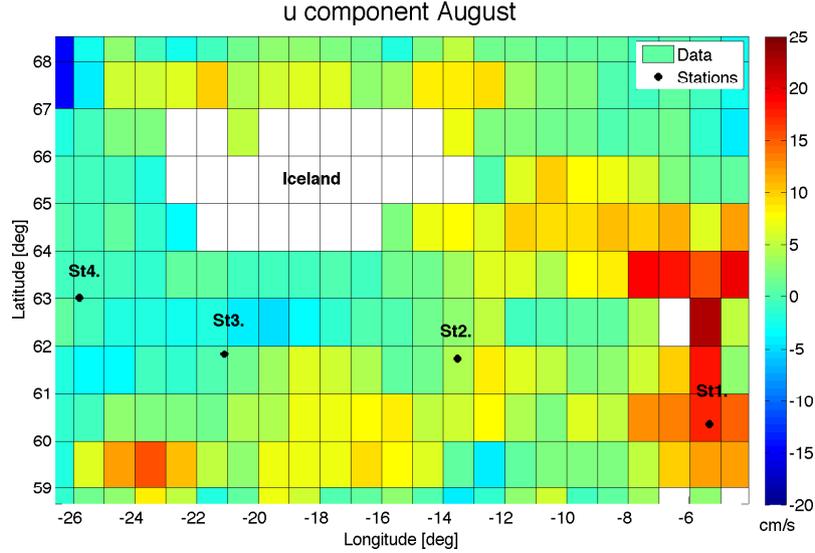


Figure 7: Pcolor plot of the reference velocity component u_{data} of the August velocity field. The data is from DAC. The positions of the stations are included.

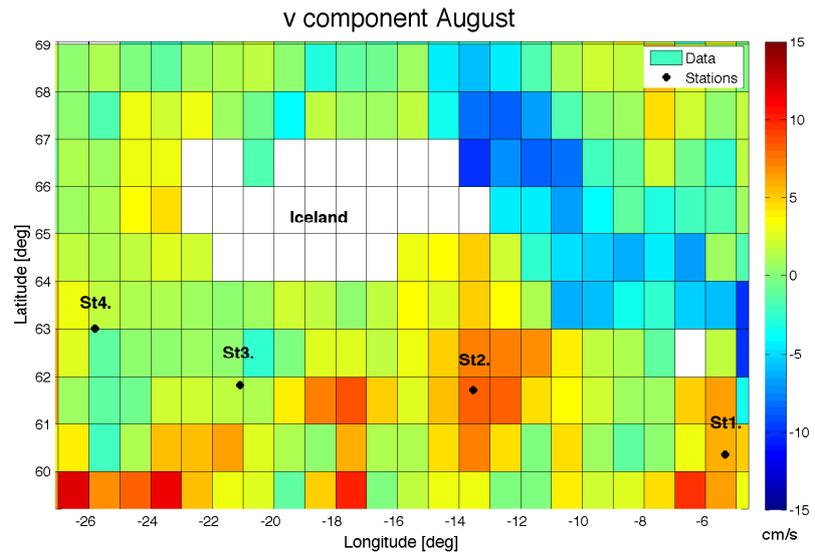


Figure 8: Pcolor plot of the reference velocity component v_{data} of the August velocity field. The data is from DAC. The positions of the stations are included.

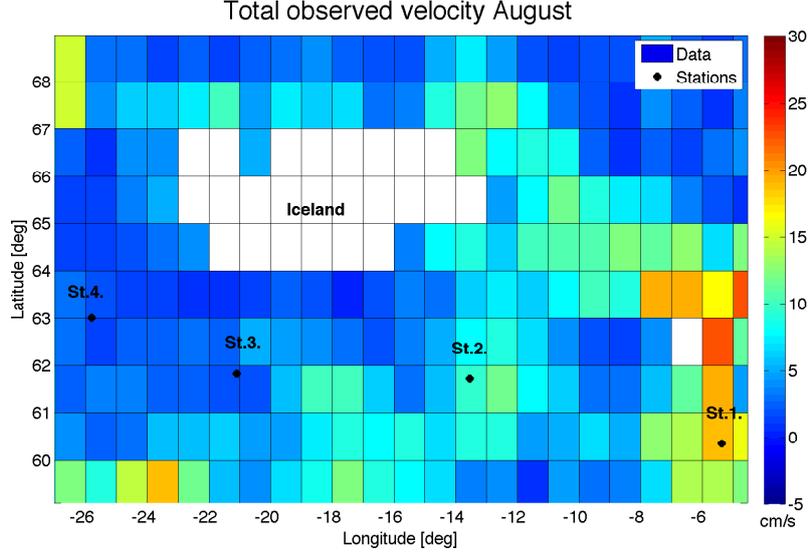


Figure 9: Pcolor plot of the total observed reference velocity $|V_{data}|$ field in August. The data is from DAC. The positions of the stations are included.

The errors on \bar{u}_{data} and \bar{v}_{data} are calculated as an average along the sailing path. Those errors are used in the error propagation formula [John R. Taylor 1982][12] to estimate the error on \bar{V}_{data} .

$$\delta\bar{V}_{data} = \sqrt{\left(\frac{\bar{u}_{data}}{|\bar{V}_{data}|}\delta\bar{u}_{data}\right)^2 + \left(\frac{\bar{v}_{data}}{|\bar{V}_{data}|}\delta\bar{v}_{data}\right)^2} \quad (19)$$

The following figure shows $|\bar{V}_{data}|$ and its corresponding error between each station pair.

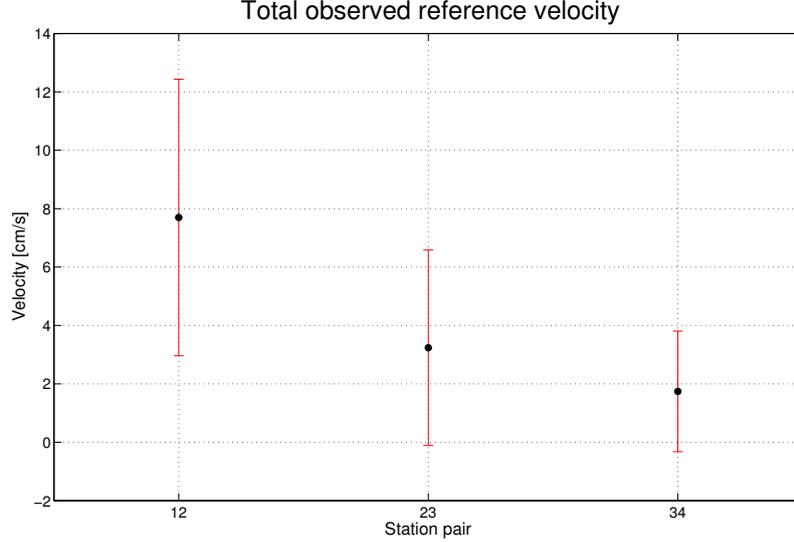


Figure 10: The total observed mean reference velocity $|\bar{V}_{data}|$ for each station pair taken from DAC. Error bars on the velocities are included.

$|\bar{V}_{data}|$ between each station pair is.

$$\begin{aligned} |\bar{V}_{data12}| &= 7.70 \pm 4.74 \text{ cm/s} \\ |\bar{V}_{data23}| &= 3.24 \pm 3.35 \text{ cm/s} \\ |\bar{V}_{data34}| &= 1.74 \pm 2.10 \text{ cm/s} \end{aligned} \quad (20)$$

To find v'_{abs} , a final reference velocity v'_{fin} is calculated. v'_{fin} is the projection of the given \bar{V}_{data} between two stations onto v'_{rel} in the $x'y'$ coordinate-system. The direction of v'_{rel} is perpendicular to the sailing direction

of the ship. The sailing direction of the ship and the direction of v'_{rel} constitutes the $x'y'$ coordinate system which is tilted with an angle θ to the original xy coordinate system. Using simple trigonometric calculations, the inclination of the two coordinate systems can be found.

$$\cos(\theta) = \frac{L_\lambda}{L} \quad (21)$$

where L is the distance between two hydrographic stations in the $x'y'$ coordinate-system and L_λ is the longitudinal distance between two stations in the xy coordinate system, given by.

$$L_\lambda = \cos(\bar{\phi}) * \Delta\lambda * R \quad (22)$$

L can be found on a sphere by the formula

$$L = R\sqrt{(\Delta\phi)^2 + (\cos(\bar{\phi}) * (\Delta\lambda))^2} \quad (23)$$

Where R is the radial distance on the earth and ϕ , λ are the latitudes and longitudes.

The tilted $x'y'$ coordinate systems also creates the angle β between v'_{rel} and $\bar{\mathbf{V}}_{data}$. β is the angle, which is used to calculate the projection of $\bar{\mathbf{V}}_{data}$ onto v_{rel} . To calculate β , the directions and magnitudes of $\bar{\mathbf{V}}_{data}$ is important.

The direction of $\bar{\mathbf{V}}_{data}$ is defined by the angle α to the x axis.

$$\tan(\alpha) = \frac{\bar{u}_{data}}{\bar{v}_{data}} \quad (24)$$

Where \bar{u}_{data} and \bar{v}_{data} is the velocity components of $\bar{\mathbf{V}}_{data}$. Now, by remembering that the angle θ is due to the tilted coordinate system, it is possible to determine β .

$$\beta = |90 - \theta - \alpha| \quad (25)$$

v'_{fin} can then be determined by

$$v'_{fin} = \cos(\beta)\bar{\mathbf{V}}_{data} \quad (26)$$

v'_{fin} is used to find v'_{abs} , from the thermal wind equation, Eq. 11.

The following figure shows v'_{fin} of each station. Errors of v'_{fin} are determined by the error propagation formula, Eq. 18, to see how the errors of $\bar{\mathbf{V}}_{data}$ propagates in the calculations of v'_{fin} .

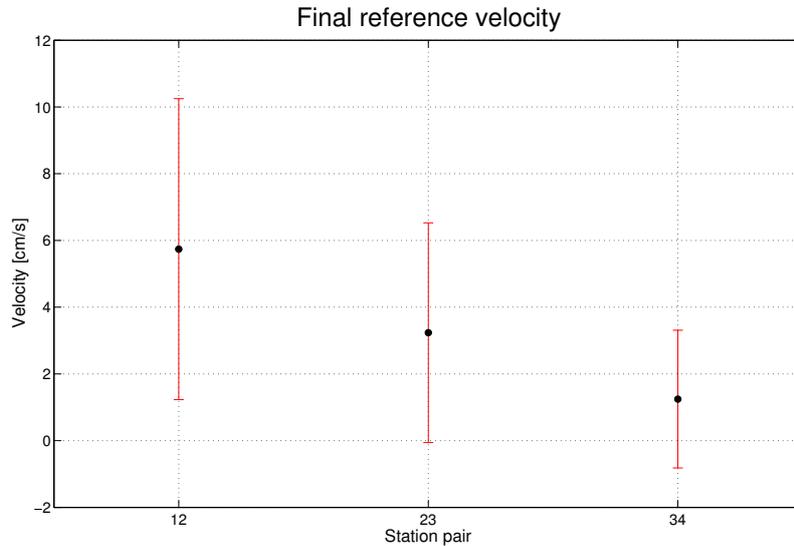


Figure 11: The final reference velocity v'_{fin} for each station taken from DAC. Error bars on the velocities are included.

Results of v'_{fin} are determined to be

$$\begin{aligned} v'_{fin12} &= 5.74 \pm 4.51 \text{ cm/s} \\ v'_{fin23} &= 3.24 \pm 3.29 \text{ cm/s} \\ v'_{fin34} &= 1.25 \pm 2.10 \text{ cm/s} \end{aligned} \quad (27)$$

v'_{abs} is then calculated by using v'_{fin} . v'_{fin} is a surface velocity, and from the definition of the thermal wind equation, Eq. 11, v'_{abs} is given by.

$$v'_{abs} = v'_{fin} - v'_{rel} \quad (28)$$

Profiles of v'_{abs} can be seen in fig. 4, 5 and 6. Now, one can look at the total transport Q_{tot} through each station pair. If Q_{tot} is large, it is a result of sparse observations made on board R/V DANA.

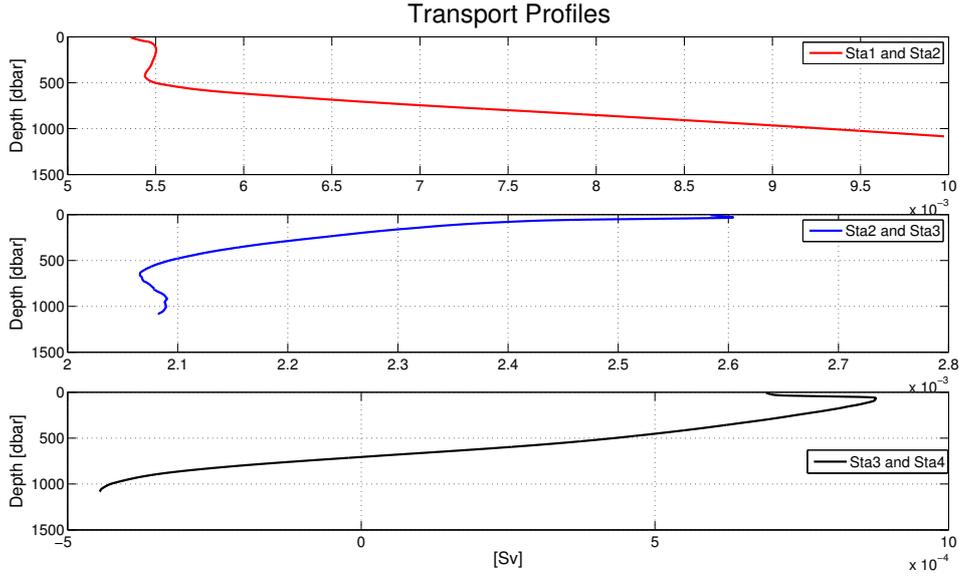


Figure 12: Geostrophic transport Q_{tot} profiles over each station pair. The sum of every transport element gives the total geostrophic transport.

The remaining hydrographic station pairs, station pair 1-3 and 2-4, can be found on appendix A. Results of the total geostrophic transport Q between each station pair is.

$$\begin{aligned} Q_{data12} &= 35.44 Sv \\ Q_{data23} &= 11.64 Sv \\ Q_{data34} &= 1.39 Sv \\ Q_{data13} &= 45.85 Sv \\ Q_{data24} &= 10.25 Sv \end{aligned} \quad (29)$$

It is now possible to look at Q_{tot} of v'_{abs} through each hydrographic station pair. This is done in two different ways. First, Q_{tot} through the line defined by the three station pairs, 1-2, 2-3 and 3-4 is considered. Secondly, Q_{tot} through a enclosed volume, pictured as a box defined by the hydrographic stations, is considered.

The following figures show the two different approaches to evaluate Q_{tot} . The hydrographic stations are plotted onto $|V_{data}|$ field from August. Arrows indicate directions of the transports.

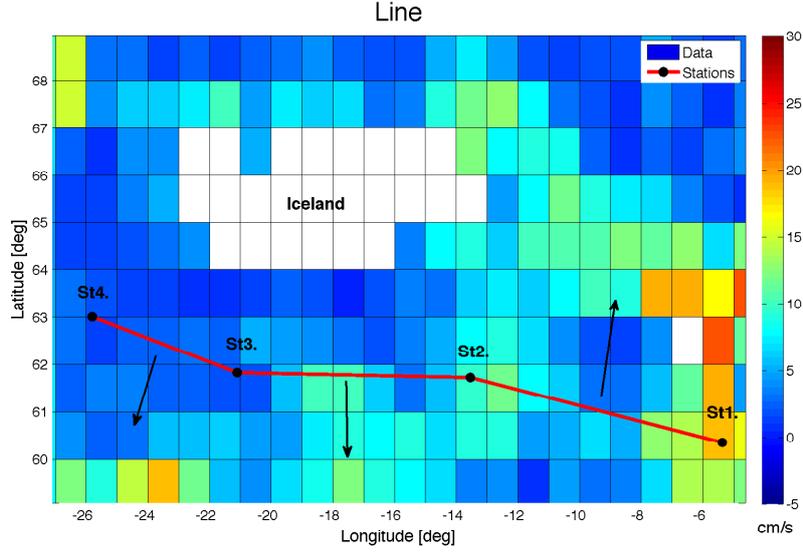


Figure 13: Geostrophic transports Q through a line defined by hydrographic stations.

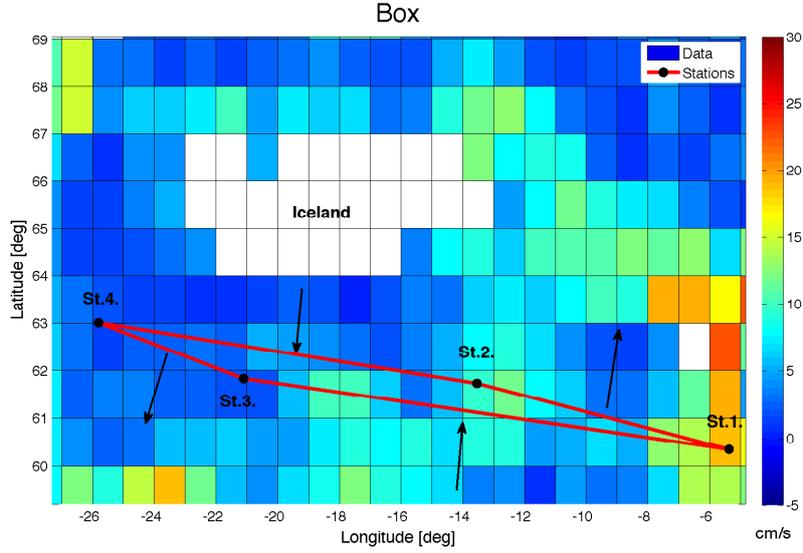


Figure 14: Geostrophic transports Q through a box defined by hydrographic stations.

Q_{tot} through the line of the hydrographic stations is,

$$Q_{linetot} = Q_{line12} - Q_{line23} - Q_{line34} = 22.40 Sv \quad (30)$$

Where positive transport is assumed to be northward and negative is southward.

Q_{tot} through the box defined by the hydrographic stations is,

$$Q_{boxtot} = Q_{box13} + Q_{box24} - Q_{box12} - Q_{box34} = 19.27 Sv \quad (31)$$

Where positive is assumed to be transport into the box and negative is assumed to be transport out of the box.

Determination of reference velocities by inverse methods

In this section a determination of reference velocities v'_b , v'_{600} and v'_{1000} are made by following the inverse methods, the box model and level of no motion. One has to remember that those reference velocities are determined only on basis of observations made on board R/V DANA, whereas the following reference velocities are theoretical, due to the lack of a known absolute velocity. The estimated reference velocities are directed on the v'_{rel} direction.

Box model method

The box model is an inverse method to estimate a geostrophic reference velocity v'_b [C. Wunsch 1996][2]. The method is based on conservation of mass in an enclosed volume defined by a Ship, which positioning hydrographic stations along the periphery of the enclosed volume, which can be seen on fig. 15.

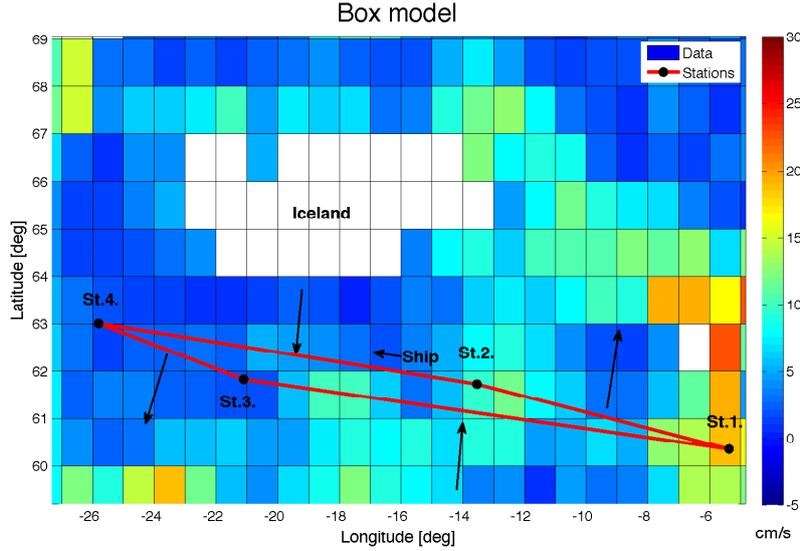


Figure 15: Enclosed volume defined by hydrographic stations. Large arrows indicate transport. Where positive is assumed in to the box. Further, the small arrow indicates the direction of the ship.

The box model uses v'_{rel} derived from the thermal wind equation Eq. 11. In this section a reference velocity v'_b is assumed as a surface velocity to compare with the reference velocity v'_{fin} from the data set. The absolute velocity from the box model v'_{ab} is given by.

$$v'_{ab} = v'_b - v'_{rel} \quad (32)$$

By making the assumption, that the total geostrophic transport Q_{tot} is zero, one can solve for v'_b .

$$Q_{tot} = \sum_{j=1}^4 \sum_q (v'_b - v'_{relj}(q)) \delta \Delta A_j(q) = 0 \quad (33)$$

Where q designates the depth interval, j is the station pair, ΔA is the differential area of the j station pair. δ is the unit normal to the enclosed volume, where positive is assumed into the volume and negative is assumed out of the volume. Further, northward velocities and transports are assumed positive and southward negative.

From the box model v'_b has been determined to.

$$v'_b = 16.82 \text{ cm/s} \quad (34)$$

v'_b is calculated from the assumption of conservation of volume. In this project a fixed total depth of 1083.6 m at all stations was used. It could be a rough estimate, because the depth varies from station to station, so the box does not contain the whole water column.

Level of no motion method

A level of no motion (lonm.) is another inverse method, that oceanographers use to find absolute velocity v'_{lonm} profiles. The method is based on an assumption that in a specific reference depth z_0 , the shear of the geostrophic velocity is zero [C. Wunsch 1996][2]. Then by subtracting the velocity from z_0 to the relative velocity profile, one obtains an approximation of an v'_{lonm} . In this paper a lonm. is chosen to be 600 m and 1000 m.

The following figure shows the profiles of v'_{lonm} obtained by the lonm. method. In the right side of the figure, profiles of v'_{lonm} based on a reference lonm. at 600 m is shown. To the left, profiles of v'_{lonm} based on a reference lonm. at 1000 m is shown.

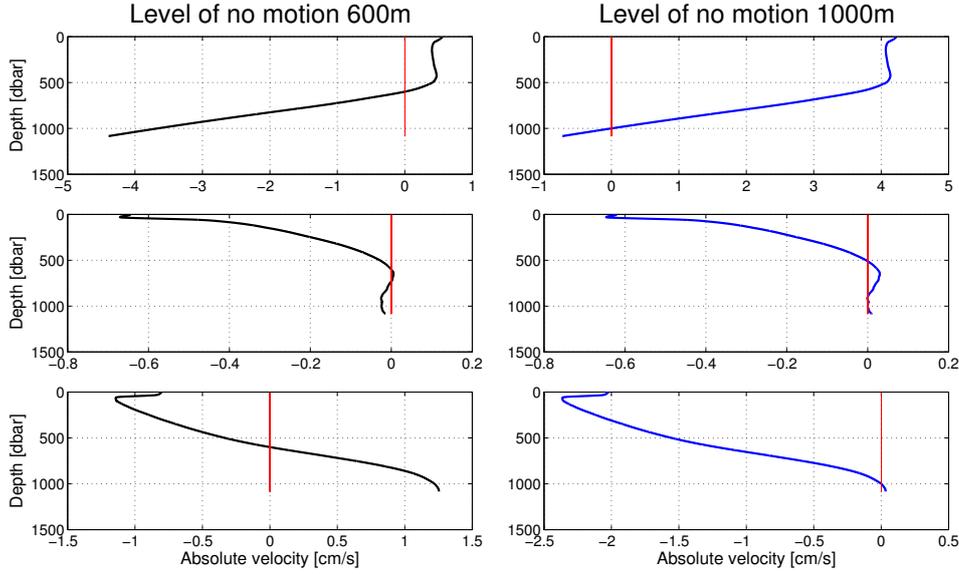


Figure 16: Absolute velocity v'_{lonm} profiles from a level of no motion at 600 m and 1000 m.

The reference velocities v'_{600} and v'_{1000} are estimated to.

Level of no motion	v'_{600}	v'_{1000}
Station 1-2	-0.56 cm/s	4.22 cm/s
Station 2-3	0.64 cm/s	0.62 cm/s
Station 3-4	0.81 cm/s	2.03 cm/s

Table 2: Reference velocities from level of no motion at 600 m and 1000 m.

One can look at the total geostrophic transport Q_{tot} through all hydrographic stations to see how well this method estimates a reference velocity. An appropriate estimation requires a low total transport through all hydrographic stations. The transport through each station pair is.

	Station pair	Total transport
Level of no motion 600m		
	Station 1-2	$Q_{600_{12}} = -3.77$ Sv
	Station 2-3	$Q_{600_{23}} = -0.50$ Sv
	Station 3-4	$Q_{600_{34}} = -0.10$ Sv
	Station 1-3	$Q_{600_{13}} = -4.26$ Sv
	Station 2-4	$Q_{600_{24}} = -0.55$ Sv
Level of no motion 1000m		
	Station 1-2	$Q_{1000_{12}} = 14.64$ Sv
	Station 2-3	$Q_{1000_{23}} = -0.39$ Sv
	Station 3-4	$Q_{1000_{34}} = -3.69$ Sv
	Station 1-3	$Q_{1000_{13}} = 14.25$ Sv
	Station 2-4	$Q_{1000_{24}} = -4.10$ Sv

Table 3: Total geostrophic transport through each station pair

Now the total geostrophic transport through all stations can be determined using the two approaches illustrated in figure 13 and figure 14.

First the total geostrophic transport through the line at a reference level at 600 m

$$Q_{600line\text{tot}} = Q_{600line12} - Q_{600line23} - Q_{600line34} = -3.21Sv \quad (35)$$

Where northward transport is assumed positive and southward transport negative. Further, the total geostrophic transport through all stations using the box method at a reference level at 600 m is

$$Q_{600box\text{tot}} = Q_{600box13} + Q_{600box24} - Q_{600box12} - Q_{600box34} = -0.98Sv \quad (36)$$

Where transport into the box is assumed positive and transport out from the box is negative. Using the same procedure, the determinations for 1000 m for the line is,

$$Q_{1000line\text{tot}} = Q_{1000_{12}} - Q_{1000_{23}} - Q_{1000_{34}} = 18.72Sv \quad (37)$$

And for the box,

$$Q_{1000box\text{tot}} = Q_{1000_{box13}} + Q_{1000_{box24}} - Q_{1000_{box12}} - Q_{1000_{box34}} = -0.79Sv \quad (38)$$

Comparison of estimated reference velocities

In this section comparisons of the reference velocities determined by inverse methods and the reference velocity v'_{fin} from the data set [Lumpkin & Johnson 2013][4] are made. On Table 4, one can see the missing errors on the determined velocities, which is due to the unknown uncertainties on the instruments. The uncertainties on the instruments are much smaller than the uncertainties on the v'_{fin} from the data set due to the high accuracy of the instruments [John A. Knaus 1997][6]. As a result, the uncertainty on v'_{fin} is only based on its own uncertainty.

Dataset	Reference velocity	Magnitude
	v'_{fin12}	5.74 ± 4.51 cm/s
	v'_{fin23}	3.24 ± 3.29 cm/s
	v'_{fin34}	1.25 ± 2.10 cm/s
Box model		
	v'_b	16.82 cm/s
Level of no motion 600m		
	$v'_{600_{12}}$	0.56 cm/s
	$v'_{600_{23}}$	-0.64 cm/s
	$v'_{600_{34}}$	-0.81 cm/s
Level of no motion 1000m		
	$v'_{1000_{12}}$	4.22 cm/s
	$v'_{1000_{23}}$	-0.62 cm/s
	$v'_{1000_{34}}$	-2.02 cm/s

Table 4: Comparison of v'_{fin} from the dataset to the calculated reference velocities using the different oceanographic inverse methods

6 Discussion

In this project only four hydrographic stations were used to compute relative velocity v'_{rel} profiles. The distances between the stations range from 275 km to 863 km, and more accurate calculations of v'_{rel} could be found, if one had a larger number of hydrographic stations separated by shorter distance.

In this paper a flat-bottom ocean was assumed, which means that a depth was fixed at all stations at 1083.8m and the surface was fixed at 6.6m depth to compute v'_{rel} profiles. These assumptions will inevitably bias the estimated transport profile due to the unknown water transport beneath the fixed depth. This results in a non zero conservation of volume of the transport determined from a reference velocity v'_{fin} from the data set. In this project, only the geostrophic velocities are considered. The real velocity in the ocean is a sum of a geostrophic and an ageostrophic velocity. The geostrophic velocity is a sum of baroclinic and barotropic

velocity components. In this paper, only the baroclinic fluid due to the thermal wind equation is considered. Both the ageostrophic and the total geostrophic part has to be included in the calculations, to obtain a total transport Q_{tot} of zero.

In figure 4, one can see the absolute velocity v'_{abs12} between station 1 and 2. It appears that v'_{abs12} increases with depth and reaches a velocity of 10 *cm/s* at 1000 m. This may be explained by the use of the reference velocity v'_{fin} from the data set. The reference velocity is a long period average of velocities measured by drifters. There is a large uncertainties on the reference velocities, see Eq. 27, which is due to the interpolation of the drifters, making them arranged in an array. The reference velocity may not completely correlates with the thermal wind derived from measurements made by Dana, that particular day.

By looking at the relative velocity profiles on fig. 4, fig. 5 and fig. 6, one can see, the relative velocities v'_{rel23} and v'_{rel34} increases from the surface to 500 m and decreases to 1000 m, where both of them are nearly stable. Looking at v'_{rel12} , there has a different pattern in its profile. v'_{rel12} follows the same pattern as v'_{rel23} and v'_{rel34} , but increases from 500 m to a depth of 1000 m, which is different from v'_{rel23} and v'_{rel34} . Station 1 is taken just south of the Greenland-Scotland Ridge. Here, the dense saline overflow water originating from the north characterizes the area. When the overflow water passes the ridge, it begins to flow down hill, while affected by gravity and frictional forces. Recalling that the thermal wind relations are derived from geostrophic and hydrostatic balance. One might say, beneath 1000 m some more complex processes might take place between station 1 and 2. Further, it can be seen, that the choice of reference of no motion at 600 m or 1000 m, have a larger impact on the corresponding reference velocity between station 1 and 2 than between the other stations.

One can see the uncertainties on v'_{fin} . There are, however, no uncertainties on the estimated reference velocities due to less information. The box model estimates only one reference velocity v'_b . One could have used more complex systems to estimate a specific reference velocity between all stations. This would cause in an equation of four unknowns, which requires more detailed assumptions.

In this paper two approaches are used to determine Q_{tot} . First, Q_{tot} through a line and secondly, Q_{tot} in and out of a box. It is obvious, that in this situation the line method is less accurate than the box method. The line does not enclose a volume, which is a requirement of the conservation of volume. If hydrographic stations were made in a line across the North Atlantic from Scotland to Greenland, the line would have enclosed a volume, assuming that all the water in the Nordic seas had to pass this line.

To investigate which of the two inverse methods is estimating the best values of v'_{fin} , a comparison was made, see table 4. The following figure 17 illustrates the accuracy of the different reference velocity estimates determined from inverse methods.

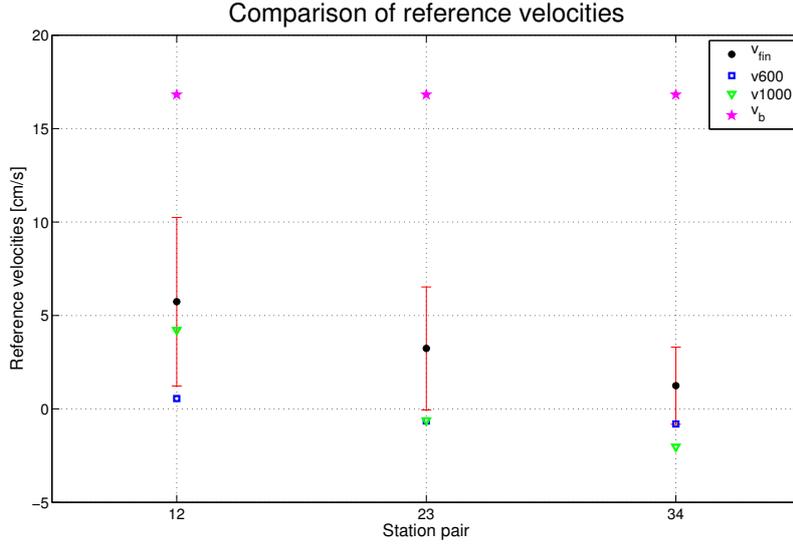


Figure 17: Figure shows the reference velocity v'_{fin} taken from the data set. Errors of v'_{fin} are plotted. Further, estimates of reference velocities of a reference level of no motion at 600m and 1000m are included.

Estimates closest to the value of v'_{fin} is the most accurate. The estimate from the box model v'_b is the reference velocity which provide conservation of volume in the box. It is difficult to compare this one value to v'_{fin} from each station. If more information were available, more complex calculations could have been done to compute reference velocities between each of the hydrographic stations.

In this project estimates of reference velocities determined from inverse methods have been compared to a reference velocity v'_{fin} from a data set. General, one would have compared the estimates to an observed velocity measured by an ADCP, which had resulted in a more actual comparison.

7 Conclusion

In this manuscript, I have demonstrated the estimation of geostrophic surface velocities using two different inverse methods, the box model and the level of no motion method. My findings are based on data from four CTD casts made in the most northern part of the Iceland-Basin and the Scotland Shetland Channel, August 27th to September 2nd 2012. A Surface reference velocity $v'_b = 16.82 \text{ cm/s}$ was estimated from the box model. Further, surface reference velocities were estimated between the hydrographic stations by the level of no motion at 600 m and 1000 m. The best estimate of the level of no motion method at 600 m was, $v'_{600_{34}} = -0.81 \text{ cm/s}$, between station 3 and 4, the best estimate of the level of no motion at 1000 m was, $v'_{1000_{12}} = 4.22 \text{ cm/s}$, between station 1 and 2 as seen on fig. 17. The level of no motion method deviates $[0.3\sigma; 1.6\sigma]$, to the reference velocities from GDP, while estimates from the Box model method deviates much more significantly $[2.5\sigma; 7.5\sigma]$.

In conclusion, the level of no motion method consistently performed better than the box model method. Making this method the preferable tool for estimating geostrophic flow profiles from a sparse set of measurements.

References

- [1] K.M Cuffey and W.S.B. Paterson. *The Physics Of Glaciers*, Fourth Edition. Elsevier, Inc.,2010
- [2] Carl Wunsch. *The Ocean Circulation Inverse Problem* chapter 2.3. Cambridge University Press 1996
- [3] Stephen Pond & George L. Pickard. *Introductory Dynamic Oceanography* chapter 8. Pergamon Press. First Edition 1978.
- [4] Lumpkin, R. and G. C. Johnson, 2013: Global Ocean Surface Velocities from Drifters: Mean, Variance, ENSO Response, and Seasonal Cycle. *J. Geophys. Res.-Oceans*, 118, in press, doi:10.1002/jgrc.20210.
- [5] John M. Wallace & Peter V. Hobbs. *Atmospheric Science, An introductory survey*, Second Edition. Elsevier Academic Press, 2006
- [6] John A. Knaus. *Introduction to Physical Oceanography*, Second Edition p. 104. Waveland Press Inc., 2005
- [7] James R. Holton. *An Introduction to Dynamic Meteorology*, Fourth Edition. Elsevier Academic Press, 2004. Chapter 3.4, Eq. (3.31)
- [8] James R. Holton. *An Introduction to Dynamic Meteorology*, Fourth Edition. Elsevier Academic Press, 2004
- [9] Lumpkin, R. and M. Pazos, 2006: Measuring surface currents with Surface Velocity Program drifters: the instrument, its data, and some recent results. Chapter two of *Lagrangian Analysis and Prediction of Coastal and Ocean Dynamics (LAPCOD)* ed. A. Griffa, A. D. Kirwan, A. J. Mariano, T. Ozgokmen, and T. Rossby.
- [10] Rio, M.-H., and F. Hernandez, High-frequency response of wind-driven currents measured by drifting buoys and altimetry over the world ocean, *J. Geophys. Res.*, 108(C8), 3283, doi:10.1029/2002JC001655, 2003.
- [11] McDougall, T.J. and P.M. Barker, 2011: *Getting started with TEOS-10 and the Gibbs Seawater (GSW) Oceanographic Toolbox*, 28pp., SCOR/IAPSO WG127, ISBN 978-0-646-55621-5.
- [12] John R. Taylor. *An Introduction To Error Analysis, The Study Of Uncertainties in Physical Measurements*, Second Edition. University Science Books, 1997

Appendix

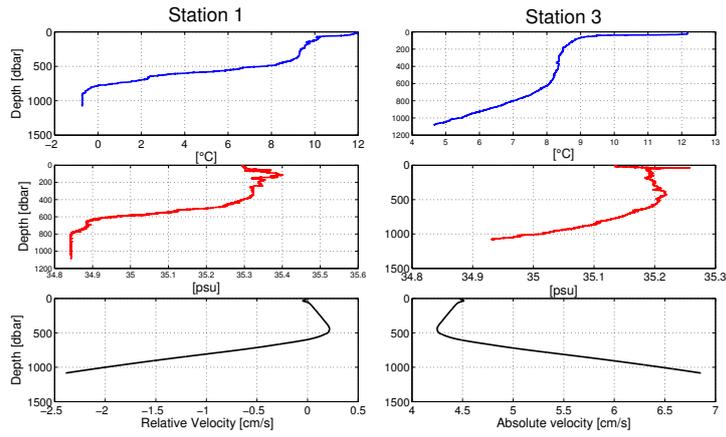


Figure 18: Relative and absolute velocity profile, calculated from temperature and salinity distributions for station 1 and 3

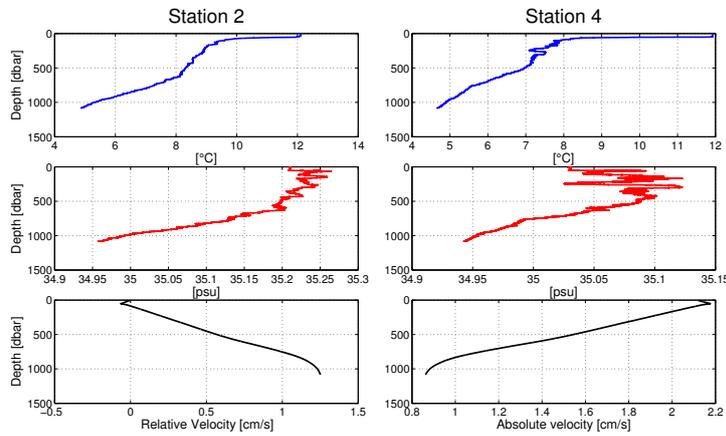


Figure 19: Relative and absolute velocity profile, calculated from temperature and salinity distributions for station 2 and 4

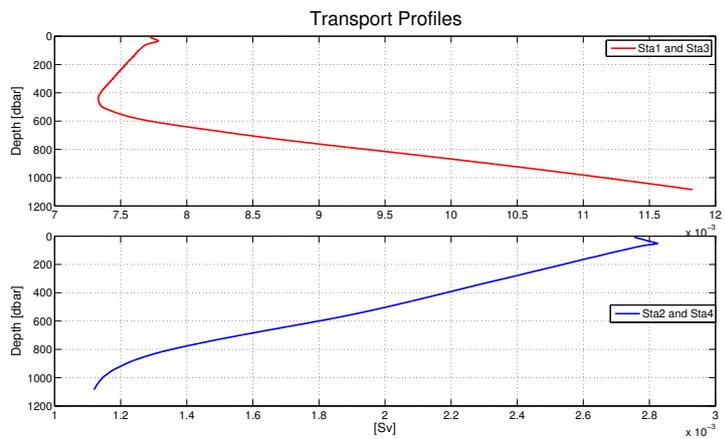


Figure 20: Geostrophic transport profiles for Station pair 1-3 and station pair 2-4.