Observation of Wave Propagation in Stationary Light

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Abstract

An atomic medium can serve as quantum memory for light in the interface of light and atoms. The property of the atomic medium as quantum memory is examined in this thesis, where the atomic ensemble consists of Λ-type atoms. It is acted on by a quantum light field and a classical control field, where the latter maps the mode of the input light to the state of the atoms. Thus, the state of the light is stored in the atomic medium.

To derive the output field of storage the equations of motion are solved analytically in the adiabatic limit. The equations of motion have been developed in a paper [1] for the beam splitter type interaction of light and an atomic medium, which is the examined interaction type. The retrieval process has already been studied in great detail, therefore, in this thesis the focus is on storage of the light and propagation of the stored wave.

The output field of storage is determined to describe the stored wave in terms of the input field. This enables us to choose parameters of the input field, which yields the desired characteristics of memory. The shape of the stored wave impose a condition on the frequency of the input light, since the frequency must be small to minimise the attenuation of the output field of storage.

The input parameters are chosen to attain the possibility of localising the stored wave at different times inside the ensemble. The shape of the stored wave changes in time, since after storage, the wave propagates and is broadening due to a time dependence of the centre and width of the stored wave. The requirement of distinguishable waves inside the ensemble imposes a condition on the optical depth of the atomic medium to be large. This condition is supported by a numerical solution to the time dependent stored wave. Furthermore, the detuning of the light frequency and the frequency of the atomic transition needs to be much greater than 1/2 to decrease broadening of the wave while increasing the distance it is propagating. Thus, conditions of the input field and the atomic medium are derived to achieve an optimised shape of the stored and propagating wave.

Contents

1 Introduction

This thesis is concerned with the interface of light and an atomic ensemble. The coupling of light and atoms can be realised with different approaches, eg. Electromagnetically induced transparency and off-resonant Raman processes. These techniques cause stationary light for optically dense media, since this interface and coupling stores the light in the medium, which makes the atomic medium serve as quantum memory for the light field.

In research papers, photon storage in atomic media is well described both in the case of a cavity and free space [1] [2] [3]. They discuss how to obtain an optimal retrieval efficiency by choosing the shape of the input light. This work expands on the former research on stationary light in a Λ-type atomic ensemble by looking into the conditions of the input light field and the atomic medium for storage of light and wave propagation inside the ensemble.

After storage of the light pulse, the wave packet propagates inside the atomic ensemble until it is retrieved by interaction of a classical control field. The requirement of being able to localise the stored wave packet imposes conditions on the shape of the input field. These conditions are examined in detail analytically and compared with the numerical solutions from the article this work is based on [1].

As the former research shows [1], a necessary condition for storage is a large optical depth for the atomic medium, which reduces loss during storage. The examination of the propagating wave packet yields conditions on the dispersion relation that applies for the Λ-type atomic medium [4], since it is required that the wave packet can be localised after propagation.

This thesis begins with an overview of the considered system, a Λ-type atomic medium acted on by a quantum field and a classical control field, to introduce the reader to the main ideas behind this thesis. Second, the equations of motion are given and discussed. Finally, the main part of the theoretical work of the thesis, based on the equations of motion, is described in the two chapters Storage of Input Light and Propagation of Stored Wave in Atomic Ensemble.

2 Interface of Light and Atomic Medium

The interface of light and an atomic medium can be divided into three parts: storage of the light in the atomic ensemble, propagation of the stored wave inside the ensemble and retrieval of the light. The atomic medium, consisting of many multilevel atoms, is acted on by a quantum light field and a classical control field. The fields interact only with the three levels depicted in Fig. 2.1, so the remaining levels of the medium are disregarded. The ensemble consists of atoms with the Λ-type atomic level scheme (see Fig. 2.1). This is the type of atoms that are used for EIT memory of light.

The states $|g\rangle$ and $|s\rangle$ are the stable ground states of the system which couple to the excited state $|e\rangle$ by the two interaction light fields, the quantum field (described

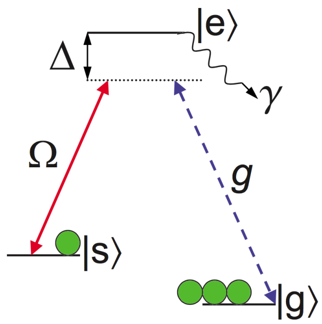

Figure 2.1: **Level scheme of Λ-type atomic medium**[3] The atoms are prepared in the ground state $|g\rangle$. The quantum field couples $|g\rangle$ and $|e\rangle$ with frequency g. The $|e\rangle - |s\rangle$ transition is is acted on by a classical control field with frequency Ω. Both fields have a detuning, Δ, with respect to state $|e\rangle$.

by $\mathcal{E}(z,t)$) with frequency g and the strong classical control field with frequency Ω, respectively. Since the fields interact with not just a single atom, but an atomic ensemble, the 3-level medium has collective states, shown in Fig. 2.1, which are described by the spin wave operator $\mathcal{S}(z,t)$. The frequencies of the light fields are detuned from resonance with the atomic transition frequency with common detuning, Δ. The only considered decay rate is the polarisation decay rate, γ, while the decay of the spin wave is neglected, γ_s, since the ratio γ_s/γ is negligible.

The transition between $|g\rangle$ and $|s\rangle$ is dipole forbidden, meaning that the $|g\rangle - |s\rangle$ transition is via the excited state $|e\rangle$. The atoms are prepared in the ground state coupled to the quantum field by optical pumping, and the light is stored when atoms are in the ground state $|s\rangle$. The state of the light is stored, since the classical control field maps the state of the input field to the state of the spin wave. At a later time, the light can be retrieved by mapping the spin wave state back to the quantum light state by interaction of the classical control field. In this storage and retrieval process, the atomic ensemble serves as quantum memory for the light.

The quantum memory can occur by using the Electromagnetically Induced Transparency (EIT) technique for storage of light inside the atomic ensemble. The EIT approach reduces the group velocity of the light, and by adiabatically turning of the control field, stationary light is developed. The reduction of the group velocity occurs, since an induced transparency window appears, where there is no absorption for a narrow range of frequencies.[5]

The quantum field acting on the $|g\rangle - |e\rangle$ transition is a quantised electric field, which is described by the annihilation and creation operator. The electromagnetic field is a classical function and the Hamiltonian of the field is used to derive the quantised field, $\mathcal{E}(z,t)$, which is an operator, in terms of the annihilation and creation operator.

During storage of light, the characteristics of the state of the stored quantum light field change due to the interaction. These changes are dependent on the parameters of the input field and the atomic medium and impose conditions on the parameters for storing the light in the atomic ensemble. It is these changes that are examined in this thesis to be able to describe the process and determine the possible values of the parameters for an experimental set-up.

3 Equations of Motion

To find the equations of motion for the operators, the Heisenberg equation of motion is used by calculating the commutator of the operators with the Hamiltonian of the system. The Heisenberg equation of motion in the Heisenberg picture is $i\hbar\frac{da(t)}{dt} = [H, a(t)]$, where the operator a is time dependent since it is given in the Heisenberg picture in which the states are time independent. This is in contrast to the Schrödinger picture with time independent operators and time dependent states[6]. The total Hamiltonian for the system is described by the sum of Hamiltonians for the light, for the atoms and for the interaction. The latter is the beam-splitter type Hamiltonian, since this type of interaction maps the state of light and atoms. The equations of motion are derived in the mainly used article in this paper[1] and the dimensionless equations of motion are given by

$$\frac{\partial}{\partial z}\mathcal{E}(z,t) = i\sqrt{d}\mathcal{P}(z,t) \tag{3.1a}$$

$$\frac{\partial}{\partial t}\mathcal{P}(z,t) = -(1+i\Delta)\sqrt{d}\mathcal{P}(z,t) + i\sqrt{d}\mathcal{E}(z,t) + i\Omega\mathcal{S}(z,t) \tag{3.1b}$$

$$\frac{\partial}{\partial t}\mathcal{S}(z,t) = i\Omega\mathcal{P}(z,t) \tag{3.1c}$$

where $\mathcal{E}$, $\mathcal{P}$ and $\mathcal{S}$ all are complex functions of (z,t). $\mathcal{E}(z,t)$ is the electric field operator, $\mathcal{S}(z,t)$ is the spin wave operator and $\mathcal{P}(z,t)$ is the polarisation operator, which is the internal state operator between the states $|g\rangle$ and $|e\rangle$.

The operators are treated as complex numbers, although they describe a quantum field coupled to the transition $|g\rangle$ to $|e\rangle$. This can be done since the operator equations of motions give complex number equations if the expectation values of the operator equations are evaluated to obtain Eq. 3.1 with coherent input states. Coherent states $|\alpha\rangle$ are eigenstates of the annihilation operator, a, with eigenvalue α: $a\,|\alpha\rangle = \alpha\,|\alpha\rangle$. Although coherent states are not eigenstates of the creation operator, the relation $\langle\alpha|\,a^\dagger = \langle\alpha|\,\alpha^*$ is satisfied. Since the coherent states are defined as linear superpositions of the photon number states, any input mode can be expanded to a set of coherent states, hence the complex number equations of motion 3.1 can be obtained.[1] [6]

To find this set of equations, the dipole approximation and the rotating-wave approximation are used, time is given by $t' = t - z/c$ to introduce a co-moving frame, and the operators, time and the coordinate, z, are rescaled to give dimensionless equations. The detuning from resonance and the frequency of the classical field are rescaled with a factor of the decay rate according to $\tilde{\Delta} = \Delta/\gamma$ and $\tilde{\Omega} = \Omega/\gamma$ to obtain dimensionless equations, and the tilde is omitted in the equations of motion. The wavelength of the light field is much larger than the size of the atoms, hence the dipole approximation is valid. The quantum field and the classical field are assumed to have frequencies near the atomic transition frequency for transitions between state $|g\rangle$ and $|e\rangle$ and between $|s\rangle$ and $|e\rangle$, respectively. In the rotating-wave approximation, the rapidly oscillating terms in the Hamiltonian are neglected because the light fields are near resonance with the atomic transition frequencies. The optical depth of the medium $d = g^2NL/(\gamma c)$ is introduced, with N the number of atoms, L the length of the medium, γ the polarisation decay rate due to spontaneous emission and c the speed of light. Decay is assumed to be in the direction perpendicular to the atomic ensemble, since reabsorption of emitted photons is neglected. This approximation is good since for incoming photons, the fraction of spontaneously emitted photons is decreasing with increasing optical depth[1], and a large optical depth is considered in the thesis as explained in chapter 5.2.

The atomic medium is approximated to a one-dimensional system to be able to treat one-dimensional equations of motion by imposing the paraxial approximation.

4 Storage of Input Light

In the preceding section, the equations of motion are defined by Eq. 3.1. These are solved analytically for the output field $\mathcal{S}_{out}(z)$ to describe storage of the light. To solve this set of differential equations, they are transformed using Fourier and Laplace transform in time and space, respectively.

First, this set of differential equations are transformed with a Fourier transform in time defined by[7]

$$\tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \tag{4.1}$$

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{f}(\omega) e^{i\omega t} dt \tag{4.2}$$

to describe the operators in the frequency domain.

$$\frac{\partial}{\partial z}\mathcal{E} = i\sqrt{d}\mathcal{P} \tag{4.3a}$$

$$-i\omega\mathcal{P} = -(1+i\Delta)\mathcal{P} + i\sqrt{d}\mathcal{E} + i\Omega\mathcal{S} \tag{4.3b}$$

$$-i\omega\mathcal{S} = i\Omega\mathcal{P} \tag{4.3c}$$

where the operators are now functions of (z,ω).

Solving for $\mathcal{E}(z,\omega)$ the differential equation of the light field in the frequency-space domain is

$$\frac{\partial}{\partial z}\mathcal{E}(z,\omega) = ik(\omega)\mathcal{E}(z,\omega) \tag{4.4}$$

where the wave vector $k(\omega)$ is introduced

$$k(\omega) = \frac{d}{-i + \Delta - \omega + \frac{\Omega^2}{\omega}} \tag{4.5}$$

The Fourier transform of the set of Eqs. 3.1 yields the spin wave operator written as a function of the light operator by

$$\mathcal{S}(z,\omega) = -\frac{\Omega}{\sqrt{d}} \frac{k(\omega)}{\omega} \mathcal{E}(z,\omega) \tag{4.6}$$

The aim is to find $\mathcal{S}(z,t)$ described by the input light field, since mapping of the light mode to the spin wave mode is described by the spin wave output field, $\mathcal{S}_{out}(z)$.

4.1 Input Light Field

A Laplace transform is made of Eqs. 4.4 and 4.6 to describe the spin wave operator as a function of the incoming light field. The transform with respect to z used, is:

$$\tilde{f}(s) = \int_0^\infty e^{-isz} f(z) dz \tag{4.7}$$

$$f(z) = \frac{1}{2\pi} \int_{i\epsilon-\infty}^{i\epsilon+\infty} e^{isz} \tilde{f}(s) ds \tag{4.8}$$

where the inverse transform (Eq. 4.8) is an integral over the real axis, since the approximation $\epsilon \ll 1$ is made. The transformed coordinate s is used to avoid confusion with $k(\omega)$, since k is the standard use of the transformed spacial coordinate.

The transform of Eq. 4.4 is derived using partial integration of the derivative

$$\mathcal{L}(\frac{\partial \mathcal{E}}{\partial z}) = \int_0^\infty e^{-isz} \frac{\partial}{\partial z} \mathcal{E} dz = [e^{-isz}\mathcal{E}]\Big|_0^\infty - (-is) \int_0^\infty e^{-isz} \mathcal{E} dz = \mathcal{E}(0,\omega) + is\mathcal{E}(s,\omega) \tag{4.9}$$

where the imaginary part of s is finite and not equal zero to yield $e^{-is\infty}\mathcal{E}(\infty,\omega) = 0$, and $\mathcal{E}(0,\omega) = \mathcal{E}_{in}(\omega)$. From Eqs. 4.4 and 4.9 we now obtain

$$ik\mathcal{E}(s,\omega) = \mathcal{E}_{in}(\omega) + is\mathcal{E}(s,\omega) \tag{4.10}$$

which relates the light as a function of the transformed coordinates (s,ω) to the incoming light. Rewritten in a simpler form gives

$$\mathcal{E}(s,\omega) = -\frac{i}{s-k} \mathcal{E}_{in}(\omega) = -i\pi\delta(s-k)\mathcal{E}_{in}(\omega) \tag{4.11}$$

where the approximation $\frac{1}{x-x_0} \approx \pi\delta(x-x_0)$ is used in the last step.

The spin wave operator can now be described as a function of the incoming light due to the Laplace transform of Eq. 4.6, which merely transforms the spacial coordinate of the operators, and Eq. 4.11 by

$$\mathcal{S}(s,\omega) = i\pi\frac{\Omega}{\sqrt{d}}\frac{k(\omega)}{\omega}\delta(s-k)\mathcal{E}_{in}(\omega) \tag{4.12}$$

The incoming wave of light is the normalised Gaussian function

$$\mathcal{E}_{in}(t) = \frac{1}{\sqrt{\sqrt{2\pi}\sigma}}e^{-\frac{(t-T/2)^2}{4\sigma^2}+i\delta t} \tag{4.13}$$

centred around $T/2$ with width σ. The incoming light as a function of frequency is the Fourier transform of Eq. 4.13. Since $\mathcal{E}_{in}(t)$ is a Gaussian function, the Fourier transformed $\mathcal{E}_{in}(\omega)$ is likewise a Gaussian

$$\mathcal{E}_{in}(\omega) = \sqrt{\sigma\sqrt{\frac{2}{\pi}}}e^{-\sigma^2(\omega-\delta)^2-i\frac{T}{2}(\omega-\delta)} \tag{4.14}$$

The normalisation is determined with $\int_{-\infty}^{\infty}|\mathcal{E}_{in}(t)|^2dt = \int_{-\infty}^{\infty}|\mathcal{E}_{in}(\omega)|^2d\omega = 1$.

To determine the spin wave operator, $\mathcal{S}(z,t)$, the inverse Laplace transform (Eq. 4.8) and the inverse Fourier transform (Eq. 4.2) is made of $\mathcal{S}(s,\omega)$. The inverse Laplace transform gives

$$\mathcal{S}(z,\omega) = \frac{1}{2\pi}\int_{i\epsilon-\infty}^{i\epsilon+\infty} i\pi\frac{\Omega}{\sqrt{d}}\frac{k(\omega)}{\omega}\delta(s-k)\mathcal{E}_{in}(\omega)e^{isz}ds = \frac{i}{2}\frac{\Omega}{\sqrt{d}}\frac{k(\omega)}{\omega}\mathcal{E}_{in}(\omega)e^{ik(w)z} \tag{4.15}$$

The integration from $i\epsilon-\infty$ to $i\epsilon+\infty$ is made over the real axis from $-\infty$ to $+\infty$ by neglecting the imaginary part. This is done by letting the limit of ϵ go to 0.

The inverse Fourier transform of $\mathcal{S}(z,\omega)$ is

$$\mathcal{S}(z,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\frac{i}{2}\frac{\Omega}{\sqrt{d}}\frac{k(\omega)}{\omega}\mathcal{E}_{in}(\omega)e^{ik(w)z}e^{i\omega t}d\omega \tag{4.16}$$

The output field from storage of light is the spin wave operator at time $t=T$

$$\mathcal{S}_{out}(z) = \mathcal{S}(z,t=T) = \frac{i}{2\sqrt{2\pi}}\frac{\Omega}{\sqrt{d}}\int_{-\infty}^{+\infty}\frac{k(\omega)}{\omega}\mathcal{E}_{in}(\omega)e^{ik(w)z}e^{i\omega T}d\omega \tag{4.17}$$

By expanding $k(\omega)$ for $\omega\approx\delta$, the equations of motion can be solved analytically for the output spin-wave operator to treat storage of the input light. The Taylor expansions are $k(\omega)\approx k(\delta)+\frac{dk(\omega)}{d\omega}\Big|_{\omega=\delta}(\omega-\delta)$ and $\frac{k}{\omega}\approx\frac{k(\delta)}{\delta}+\frac{d}{d\omega}\frac{k(\omega)}{\omega}\Big|_{\omega=\delta}(\omega-\delta)$

Using this expansion of $k(\omega)$ in Eq. 4.17, the integral is centred around the variable $\xi=\omega-\delta$. Solving the integral for the output field of storage gives

$$\mathcal{S}_{out}(z) = \frac{i\Omega}{2\sqrt{\sqrt{2\pi}\sigma^5 d}}[(\frac{k}{\omega})'((\frac{k}{\omega})'-\frac{T}{2})+2\frac{k}{\delta}\sigma^2]e^{i\delta T}e^{ikz}e^{-\frac{(z-z_0)^2}{4\sigma_0^2}} \tag{4.18}$$

where $(\frac{k}{\omega})'$ denotes the derivative $\frac{d}{d\omega}\frac{k(\omega)}{\omega}\Big|_{\omega=\delta}$, and the parameters z_0, σ_0 and k are all complex numbers.

$$z_0 = \frac{T/2}{k'} = \frac{T}{2}\frac{(-i\delta - \delta^2 + \Delta\delta + \Omega^2)^2}{d(\delta^2 + \Omega^2)} \tag{4.19}$$

$$\sigma_0 = \frac{\sigma}{k'} = \sigma\frac{(-i\delta - \delta^2 + \Delta\delta + \Omega^2)^2}{d(\delta^2 + \Omega^2)} \tag{4.20}$$

$$k = \frac{d}{-i - \delta + \Delta + \Omega^2/\delta} \tag{4.21}$$

where k' denotes the derivative $\frac{dk(\omega)}{d\omega}\Big|_{\omega=\delta}$.

To describe the shape of the stored light, the output field should be of the form of a purely real Gaussian function with a complex phase. $\mathcal{S}_{out}(z)$ is rewritten by denoting $z_0 = z_R + iz_I$ and $4\sigma^2 = A + iB$, where z_R is the real part of z_0 and z_I is the imaginary part of z_0. Likewise is A the real part of $4\sigma^4$ and B the imaginary part.

$$z_R = \frac{T}{2}\frac{((\Delta\delta - \delta^2 + \Omega^2)^2 - \delta^2)}{d(\delta^2 + \Omega^2)} \tag{4.22a}$$

$$z_I = \frac{-T\delta(\Delta\delta - \delta^2 + \Omega^2)}{d(\delta^2 + \Omega^2)} \tag{4.22b}$$

$$A = \frac{4\sigma^2}{d^2(\delta^2 + \Omega^2)^2}[(\Delta\delta - \delta^2 + \Omega^2)^4 - 6\delta^2(\Delta\delta - \delta^2 + \Omega^2) - \delta^4] \tag{4.23a}$$

$$B = \frac{-16\sigma^2\delta}{d^2(\delta^2 + \Omega^2)^2}[((\Delta\delta - \delta^2 + \Omega^2)^2 - \delta^2)(\Delta\delta - \delta^2 + \Omega^2)] \tag{4.23b}$$

The output field is rewritten in terms of the real numbers z_R, z_I, A, B by

$$-\frac{(z - z_0)^2}{4\sigma_0^2} = -\frac{(z - (z_R + iz_I))^2}{A + iB} = -\frac{(A - iB)(z - z_s)^2 + C + i\tilde{k}z + i\phi}{A^2 + B^2} \tag{4.24}$$

where z_s is the centre of the Gaussian and $\frac{1}{2}\sqrt{\frac{A^2+B^2}{A}}$ the width. $\frac{C}{A^2+B^2}$ is a real factor, $\frac{\phi}{A^2+B^2}$ is a phase and $\frac{\tilde{k}}{A^2+B^2}$ is a part of the real wave vector, where the other part is the real part of k (Eq. 4.21).

$$z_s = z_R + \frac{B}{A}z_I \tag{4.25}$$

$$\sigma_s = \frac{1}{2}\sqrt{\frac{A^2 + B^2}{A}} \tag{4.26}$$

$$C = (A + \frac{B^2}{A})z_I \tag{4.27}$$

$$\tilde{k} = 2(\frac{B^2}{A} + A)z_I \tag{4.28}$$

$$\phi = 2(A - \frac{B^2}{A})z_I z_R - B(\frac{B^2}{A^2} + 1)z_I^2 \tag{4.29}$$

The attenuation of the wave is given by the term ikz in Eq. 4.18, since $k(\delta)$ has an imaginary part. That results in a factor of $e^{-Im(k)z}$ in $\mathcal{S}_{out}(z)$.

$$Im(k) = \frac{d}{1 + (-\delta + \Delta + \Omega^2/\delta^2)^2} = \frac{d\delta^2}{\delta^2 + (-\delta^2 + \Delta\delta + \Omega^2)^2} \tag{4.30}$$

To decrease the loss, $Im(k)$, δ needs to be small, since the attenuation is proportional to δ^2. For $\delta \ll 1$ and $\delta\Delta \ll 1$ the loss is approximately

$$Im(k) \approx \frac{d\delta^2}{\Omega^4} \tag{4.31}$$

where it is clear that the smallest order of δ is δ^2 and hence a decrease of δ decreases the loss of the output operator. To first order in δ the centre and the width of the output spin-wave operator are

$$z_s \approx \frac{T}{2d}(2\Delta\delta + \Omega^2) \tag{4.32}$$

$$\sigma_s \approx \frac{\sigma}{d}(2\Delta\delta + \Omega^2) \tag{4.33}$$

Both z_s and σ_s have a term independent of δ.

When the light is stored in the atoms the Gaussian wave of $\mathcal{S}_{out}(z)$ propagates in time according to the dispersion relation of the system, which is described in more detail in the following chapter. Since the wave moves, it is optimal to store the wave at $z_s \approx 0.25$ and with a small width, chosen $\sigma_s \approx 0.05$, because the width increases as a function of time for the propagating wave. These properties of the stored field can be converted to the parameters of the input field. When the centre and the width is chosen $z_s = 0.25$ and $\sigma_s = 0.05$ it gives, in terms of the input field and duration time, T

$$T = \frac{d}{2(2\Delta\delta + \Omega^2)} \tag{4.34}$$

$$\sigma = \frac{d}{20(2\Delta\delta + \Omega^2)} \tag{4.35}$$

which determines the duration time as a function of the width of the input field

$$T = 10\sigma \tag{4.36}$$

The loss is given by the imaginary part of k as before mentioned. The loss can be described in terms of the centre and the width of the input field for small δ

$$z_s \approx \frac{T}{2d}\frac{d\delta^2}{Im(k)} \tag{4.37a}$$

$$\sigma_s \approx \frac{\sigma}{d}(\frac{d\delta^2}{Im(k)})^2 \tag{4.37b}$$

Which describes the attenuation in two ways

$$Im(k) \approx \frac{T\delta^2}{2z_s\Omega^2} \tag{4.38a}$$

$$Im(k) \approx \sqrt{\frac{\sigma d\delta^4}{\sigma_s\Omega^2}} \tag{4.38b}$$

When solving the Eq. 4.17 the wave vector $k(\omega)$ was expanded to first order in $(\omega-\delta)$, which means that the only loss included was due to the width of the wave. Including the second order term of δ would give an extra loss for small optical depth, which can be seen by evaluating the second order derivative, since the only term of $(\omega-\delta)^2$ in Eq. 4.17 is $e^{-\sigma^2(\omega-\delta)^2}$ and the second order derivative is the second order term in the Taylor expansion of $k(\omega)$

$$\frac{d^2k(\omega)}{d\omega^2} = \frac{2d\omega(-i\omega-\omega+\Delta\omega+\Omega^2)^2 + 2d(\omega+\Omega^2)(1+2\omega-\Delta)}{(-i\omega-\omega+\Delta\omega+\Omega^2)^3} \tag{4.39}$$

In the limit $\delta \to 0$ it gives

$$\frac{d^2k(\omega)}{d\omega^2} \approx \frac{2d(i-\Delta)}{\Omega^4} \tag{4.40}$$

The optical depth, d, can be varied by varying the frequency of the control field. This is done by keeping $\frac{\Omega^2}{d}$ constant for different optical depths, which makes $\frac{d^2k(\omega)}{d\omega^2}$ proportional to $1/d$. Thus for a large optical depth this modification of including the second order term in $k(\omega)$ in the expansion to improve the solution can be neglected.

A comparison of $\mathcal{S}_{out}(z)$ in integral form (Eq. 4.17) and the expansion of $k(\omega)$: $k(\omega) \approx k(\delta) + \frac{dk(\omega)}{d\omega}\Big|_{\omega=\delta}(\omega-\delta) + \frac{1}{2}\frac{d^2k(\omega)}{d\omega^2}\Big|_{\omega=\delta}(\omega-\delta)^2$ shows that the added $(\omega-\delta)^2$ term results in a modification of σ, because the only term of $(\omega-\delta)^2$ in $\mathcal{S}_{out}(z)$ is $e^{-\sigma^2(\omega-\delta)^2}$ originating from $\mathcal{E}_{in}(\omega)$ (Eq. 4.14). The modified σ is therefore given by

$$\sigma^2 \to \sigma^2 - \frac{i}{2}\frac{\partial^2k(\omega)}{\partial\omega^2}z \tag{4.41}$$

This modification to the output field of storage is used in the following chapter, where the analytical solution in Eq. 4.18 is compared with the numerical solution of the exact integral solution of the equations of motion in Eqs. 3.1.

4.2 Comparison of Analytical and Numerical Solution of Storage

The validity of the analytical solution, Eq. 4.18, of storage is examined by solving the equations of motion numerically. They can be solved more exact by not assuming that the frequency is near δ, thus omitting the expansion of $k(\omega)$ for ω near δ. This is possible in the adiabatic limit, which gives a more exact integral form of $\mathcal{S}_{out}(z)$ than in the preceding calculations, where the expansion of $k(\omega)$ was introduced. The adiabatic limit eliminates the optical polarisation in Eq. 3.1b. This limit requires only the condition $Td\gamma \ll 1$ to be satisfied.

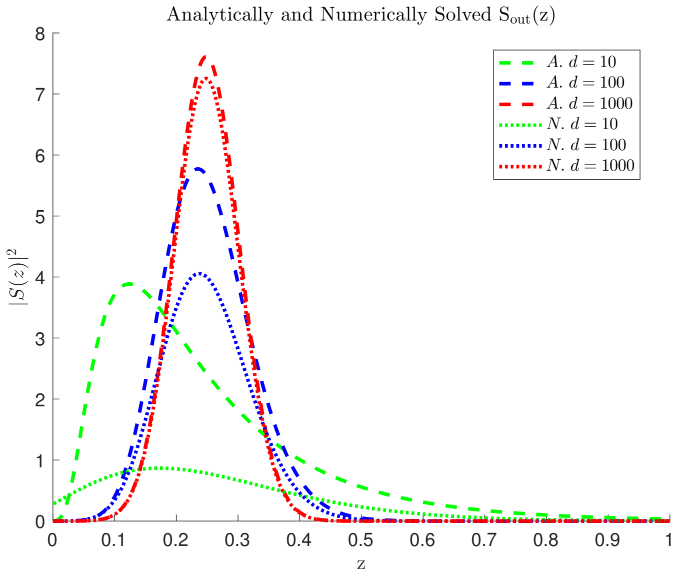

Figure 4.1: **Plot of the output field solved analytically and numerically for different optical depth, d.** Dashed lines are analytical solutions given by Eq. 4.18. Dotted lines are numerical solutions using numerical integration to solve the exact $S_{out}(z)$ (Eq. 4.42). In the analytical solutions the modified width of the Gaussian is used according to Eq. 4.41. The parameters are chosen for the case of no detuning from resonance and $\delta = 0$ to give a Gaussian wave with centre $z_s = 0.25$ and width $\sigma_s = 0.05$ for $d = 1000$.

The integral form of $\mathcal{S}_{out}(z)$, determined by solving the equations of motion in the adiabatic limit, is given by Eq. 34 in reference[1]

$$\mathcal{S}_{out}(z) = \int_0^T dt\ \Omega \frac{1}{1+i\Delta} e^{[\Omega(T-t)+dz]/(1+i\Delta)} I_0\left(2\frac{\sqrt{\Omega(T-t)dz}}{1+i\Delta}\right)\mathcal{E}_{in}(t) \tag{4.42}$$

where Ω is the constant frequency of the control field independent of (z,t) and I_0 is the zero order modified Bessel function. For large d, ie. $d = 1000$, the Bessel function can be expanded using the asymptotic series expansion for large argument $2\frac{\sqrt{\Omega(T-t)dz}}{1+i\Delta}$, with d large. For the zero order modified Bessel function the expansion to lowest order for large argument z is

$$I_0(z) \approx \frac{e^z}{\sqrt{2\pi z}} \tag{4.43}$$

The integral in $\mathcal{S}_{out}(z)$, Eq. 4.42, is solved numerically using numerical integration. For large optical depth, $d = 1000$, the expansion of the Bessel function is used to obtain finite values of $\mathcal{S}_{out}(z)$.

Fig. 4.1 shows that the analytically solved output field (Eq. 4.18) converges to the numerical solution for large optical depth, since the two solutions approach each other for $d = 1000$. That implies that the Taylor expansion of $k(\omega)$ around $\omega = \delta$ is a good approximation for a large optical depth. The condition of large optical for the analytical solution is discussed in detail in chapter 5.2.

In the analytical solution the improvement of the width given in Eq. 4.41 is included, which primarily affects output field for $d = 10$. This is seen in the figure, since the wave for $d = 10$ has a non-Gaussian tail due to the position dependent width. For $d = 100$ the analytically solved wave has a larger absolute maximum than the numerical solution, however, they are centred around the same value of z and have the same shape. The convergence of the numerical and analytical solution for large optical depth is derived analytically in chapter 5.2 by examining the parameters of the stored wave and how it propagates inside the atomic medium.

5 Propagation of Stored Wave in Atomic Ensemble

To examine the output field from storage the time evolution of $\mathcal{S}_{out}(z)$ is developed. The time evolution of a wave packet is determined by a Fourier transform in space with time dependence $e^{-i\omega t}$ [8]

$$f(z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(k) e^{i(kz - \omega(k)t)} dk \tag{5.1}$$

where $f(k)$ is the Fourier transform of $f(z,0)$, and ω is a function of k determined by the dispersion relation for the system.

To make the following calculations more clear $\mathcal{S}_{out}(z)$ is used in the form of the normalised Gaussian

$$\mathcal{S}_{out}(z) = s_0 e^{-\frac{(z-z_s)^2}{A+iB}} e^{ik_s z} \tag{5.2}$$

where s_0 is the constant of normalisation and k_s is stored wave vector

$$k_s = \frac{\tilde{k}}{A^2 + B^2} + k \tag{5.3}$$

$$s_0 = \frac{i\Omega}{2\sqrt{\sqrt{2\pi}\sigma^5 d}} [(\frac{k}{\omega})'((\frac{k}{\omega})' - \frac{T}{2}) + 2\frac{k}{\delta}\sigma^2] e^{i\delta T} e^{\frac{C+i\phi}{A^2+B^2}} \tag{5.4}$$

defined in Eq. 4.18 and 4.24.

To determine the time evolution of $S_{out}(z)$, it is Fourier transformed in space

$$\mathcal{S}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{S}_{out}(z) e^{-ikz} dz = s_0\sqrt{2}\frac{\sqrt{A+iB}}{2} e^{-(\frac{A+iB}{4})(k-k_s)^2 - iz_s(k-k_s)} \tag{5.5}$$

Eq. 5.1 is used for $\mathcal{S}(k)$ to get the time evolution of $\mathcal{S}_{out}(z)$ by adding the time dependence $e^{-i\omega(k)t}$, where ω is a function of k, and then using the inverse Fourier transform of $\mathcal{S}(k)e^{-i\omega t}$

$$\begin{aligned} \mathcal{S}(z,t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{S}(k) e^{i(kz-\omega t)} dk \\ &= \frac{s_0\sqrt{A+iB}/2}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{i(kz-\omega t)} e^{-(\frac{A+iB}{4})(k-k_s)^2 - iz_s(k-k_s)} dk \end{aligned} \tag{5.6}$$

Since $\omega(k)$'s dependence on k is determined by the dispersion relation of the medium, the integral is solved by expanding $\omega(k)$ for k near k_s to second order in $(k-k_s)$ to include the term involving dispersion.

$$\omega(k) \approx \omega(k_s) + \frac{d\omega}{dk}\bigg|_{k=k_s}(k-k_s) + \frac{1}{2}\frac{d^2\omega}{dk^2}\bigg|_{k=k_s}(k-k_s)^2 \tag{5.7}$$

The expansion is inserted in Eq. 5.6 to find the time dependent spin-wave operator after storage. It is written with $\omega(k_s) = \omega$, $\frac{d\omega}{dk}\Big|_{k=k_s} = \omega'$, $\frac{d^2\omega}{dk^2}\Big|_{k=k_s} = \omega''$ to make it more apparent.

$$\begin{aligned} \mathcal{S}(z,t) &= \frac{s_0\sqrt{A+iB}}{\sqrt{2}\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(kz-(\omega+\omega'(k-k_s)+\omega''(k-k_s)^2/2)t)} e^{-(\frac{A+iB}{4})(k-k_s)^2 - iz_s(k-k_s)} dk \\ &= \frac{s_0\sqrt{A+iB}}{\sqrt{2}\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(kz-\omega t)} e^{-(i\frac{\omega''}{2}+(\frac{A+iB}{4}))(k-k_s)^2 t} e^{-i(z_s+\omega' t)(k-k_s)} dk \\ &= \frac{s_0\sqrt{A+iB}}{\sqrt{2}\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} e^{ikz} e^{-(i\frac{\omega''}{2}+(\frac{A+iB}{4}))(k-k_s)^2 t} e^{-i(z_s+\omega' t)(k-k_s)} dk \\ &= \frac{s_0\sqrt{A+iB}}{\sqrt{2}\sqrt{2\pi}} e^{-i\omega t} \int_{-\infty}^{\infty} e^{ikz} e^{-\sigma_t^2(k-k_s)^2 t} e^{iz_t(k-k_s)} dk \end{aligned} \tag{5.8}$$

Introducing the function $\tilde{\mathcal{S}}(k;\sigma_t,z_t)$ with

$$\tilde{\mathcal{S}}(k;\sigma_t,z_t) = s_0\sqrt{2}\sigma_t e^{-\sigma_t^2(k-k_s)^2 - iz_t(k-k_s)} \tag{5.9}$$

$\mathcal{S}(z,t)$ can be rewritten in the form of the inverse Fourier transform with time dependent width and centre

$$\mathcal{S}(z,t) = s_0\sqrt{\frac{A+iB}{2}}e^{-i\omega t}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{ikz}\frac{1}{s_0\sqrt{2}\sigma_t}\tilde{\mathcal{S}}(k;\sigma_t,z_t)dk \tag{5.10}$$

where the function $\tilde{\mathcal{S}}(k;\sigma_t,z_t)$ has been introduced as the spin-wave operator defined in the form of Eq. 5.5 with $\sigma_t = \sqrt{(A+iB)/4 + i\frac{\omega''}{2}t}$ and $z_t = z_s + \omega' t$, because $\mathcal{S}(k) \to \tilde{\mathcal{S}}(k;\sigma_t,z_t)$ for $\sqrt{(A+iB)/2} \to \sigma_t$ and $z_s \to z_t$. σ_t describes the width of the Gaussian, when the wave is given in real parts and a phase. $\tilde{\mathcal{S}}(k;\sigma_t,z_t)$ is the Fourier transform of $\tilde{\mathcal{S}}(z;\sigma_t,z_t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{ikz}\tilde{\mathcal{S}}(k;\sigma_t,z_t)dk$. The integral in Eq. 5.10 can be seen as the inverse Fourier transform of $\tilde{\mathcal{S}}(k;\sigma_t,z_t)$, denoted $\tilde{\mathcal{S}}(z;\sigma_t,z_t)$, where $z_t = z_s + \omega' t$, which gives

$$\begin{aligned}\mathcal{S}(z,t) &= \frac{\sqrt{A+iB}}{2\sigma_t}e^{-i\omega t}\tilde{\mathcal{S}}(z;\sigma_t,z_t)\\ &= \frac{s_0\sqrt{A+iB}}{2\sigma_t}e^{-i\omega t}e^{-\frac{(z-(\omega' t+z_s))^2}{4\sigma_t^2}+ik_s z}\end{aligned} \tag{5.11}$$

where the following definition of $\tilde{\mathcal{S}}(z;\sigma_t,z_t)$ is used.

$$\tilde{\mathcal{S}}(z;\sigma_t,z_t) = s_0 e^{-\frac{(z-z_s)^2}{4\sigma_t^2}+ik_s z_t} \tag{5.12}$$

Since the frequency, ω, in general is complex, the Gaussian in $\mathcal{S}(z,t)$ is written with complex parameters. To define the centre, width and loss of the wave the exponential in $\mathcal{S}(z,t)$ is rewritten into a real Gaussian with a phase. $\mathcal{S}(z,t)$ is rewritten in the same way as was done for the equation of storage $\mathcal{S}_{out}(z)$ in Eq. 4.24, where the function is given with real and imaginary parts explicitly.

$$\begin{aligned}&-\frac{(z-(\omega' t+z_s))^2}{4\sigma_t^2}\\ &= -\frac{(z-(z_{tR}+iz_{tI}))^2}{A_t+iB_t}\\ &= \frac{-(A_t-iB_t)(z-z_m)^2 + (a+\frac{b^2}{a})z_{tI}^2 + 2iz_{tI}(\frac{b^2}{a}+a)(z-z_{tR}) - ib(\frac{b^2}{a^2}+1)z_{tI}^2}{A_t^2+B_t^2}\\ &= \frac{-(A_t-iB_t)(z-z_m)^2}{A_t^2+B_t^2} + \alpha + i\beta\end{aligned} \tag{5.13}$$

To express the width and centre of the Gaussian as real number the following four real parameters have been introduced

$$z_{tR} = z_s + Re[\omega']t \tag{5.14a}$$

$$z_{tI} = Im[\omega']t \tag{5.14b}$$

$$A_t = 4\sigma_s^2 - 2Im[\omega'']t \tag{5.15a}$$

$$B_t = 2Re[\omega'']t \tag{5.15b}$$

To simplify the expression the new parameters α and β have been introduced in the last step.

$$\alpha = \frac{(a + \frac{b^2}{a})z_{tI}^2}{A_t^2 + B_t^2} \tag{5.16a}$$

$$\beta = \frac{2z_{tI}(\frac{b^2}{a} + a)(z - z_{tR}) - b(\frac{b^2}{a^2} + 1)z_{tI}^2}{A_t^2 + B_t^2} \tag{5.16b}$$

The time evolution of $\mathcal{S}_{out}(z)$, $\mathcal{S}(z,t)$, after storage of light is given as a Gaussian with real width, $\frac{1}{4\sigma_f^2} = \frac{A_t}{A_t^2+B_t^2}$ and centre, $z_f = z_s + Re[\omega']t$ by

$$\mathcal{S}(z,t) = \frac{s_0\sqrt{A+iB}}{2\sqrt{(A+iB)/4 + i\omega''t/2}} e^{-i\omega t} e^{\frac{-(A_t - iB_t)(z-z_f)^2}{A_t^2+B_t^2}} e^{ik_s z + \alpha + i\beta} \tag{5.17}$$

$\mathcal{S}(z,t)$ is a Gaussian function with a time dependent overall loss due to broadening of the wave, since the width $Re[\sigma_t]$ is time dependent. It is centred around $z_f = z_s + Re[\omega']t$, which tells that the centre of the stored wave, z_s is propagated by $Re[\omega']t$ and hence dependent of time.

The time dependent width, σ_f, given in terms of the stored parameters, is

$$\sigma_f = \frac{1}{2}\sqrt{\frac{A_t^2 + B_t^2}{A_t}} = \sqrt{\frac{(2\sigma_s^2 - Im[\omega'']t)^2 + (Re[\omega'']t)^2}{4\sigma_s^2 - 2Im[\omega'']t}} \tag{5.18}$$

Since $Im[\omega'']$ is negative, the wave is broadening when propagating inside the ensemble due to the time dependence of the width.

5.1 Conditions Imposed by the Dispersion Relation

The dispersion relation, $\omega(k)$, for this system is the quadratic dispersion relation for stationary light in Λ-type atoms, which expanded for small $(\frac{k}{n_0})^2$, where n_0 is the density of atoms, is[4]

$$\omega = \frac{1}{2m}(\frac{k}{n_0})^2 \tag{5.19}$$

and the inverse of the mass is defined by[4]

$$\begin{aligned} \frac{1}{m} &= \frac{16^2\pi^2\gamma\Omega^2(\Delta - i/2)}{d^2} n_0^2 \\ &= (v_r + iv_i)n_0^2 \end{aligned} \tag{5.20}$$

where v_r is the real part and v_i the imaginary part of $1/(mn_0^2)$. The decay causing reabsorption is neglected in the definition of $1/m$ like in the preceding calculations.

This dispersion relation is determined for Λ-type atomic medium with the level scheme shown in Fig. 2.1, where the atoms are initially prepared in the ground state $|g\rangle$, and the rotating wave approximation is used to solve the equations of motions. Hence this dispersion relation applies for the system considered.

The centre of $\mathcal{S}(z,t)$, z_f is time dependent and moves with velocity $Re[\omega']$ with ω' given by

$$\omega' = \left.\frac{d\omega}{dk}\right|_{k=k_s} = (v_r + iv_i)k_s \tag{5.21}$$

If the wave vector k_s of storage is considered real, the centre moves with the real part of $1/m$

$$Re[\omega'] = v_r k_s \tag{5.22}$$

k_s can be considered real in the limit of $\delta \ll 1$, since for $\delta \to 0$, the imaginary part of k_s can be neglected, $k_s \to Re[k_s]$.

The attenuation of $\mathcal{S}(z,t)$ in time is identified with the factor $e^{-i\omega t}$, since ω is complex. As explained the wave vector of storage is chosen real, which gives an attenuation of $Im[\omega]$.

$$e^{-i\omega t} = e^{-iRe[\omega]t + Im[\omega]t} \tag{5.23}$$

shows that the loss as a function of time is $Im[\omega]t$, with $Im[\omega]$ negative according to Eq. 5.19, which is an attenuation of the wave.

To optimise the storage of light, the attenuation in time needs to be small compared to the velocity, with which the centre propagates, since the wave needs to propagate inside the ensemble with minimal loss. That imposes the condition $|Im[\omega]| \ll |Re[\omega']|$. For this condition on ω to be satisfied, the dispersion relation in Eq. 5.19 needs to satisfy

$$\left|\frac{1}{2} v_i k_s^2\right| \ll |v_r k_s| \tag{5.24}$$

For this relation to be satisfied for all choices of k_s, it confines $1/m$ by

$$\Delta \gg \frac{1}{2} \tag{5.25}$$

which is determined from $1/m$ in Eq. 5.20.

5.2 Condition on Optical Depth for Storage

The solution of the time dependent spin wave operator, Eq. 5.11, imposes a condition for storage of the light, since for storage and retrieval to be possible, the wave has to propagate inside the ensemble with width and broadening small enough to distinguish the stored and the propagated wave, ie. localisation of the wave inside the ensemble. Written in terms of the propagating centre, $\tilde{z}$, and width Δz the condition is

$$\tilde{z} \gg \Delta z \tag{5.26}$$

where $\tilde{z}$ and Δz are defined by approximations of z_f and σ_f, respectively. The conditions for minimum loss of the stored wave and the propagating wave, $\delta \ll 1$ and $Im[\frac{1}{m}] \ll Re[\frac{1}{m}]$, respectively, are imposed in the following definitions of $\tilde{z}$ and Δz

$$\tilde{z} = Re[\omega']t \tag{5.27}$$

$$\Delta z = \frac{1}{2}\sqrt{4\sigma_s^2 + \frac{Re[\omega'']^2 t^2}{\sigma_s^2}} \tag{5.28}$$

Δz has to be minimized to satisfy the condition $\tilde{z} \gg \Delta z$. The minimal value for Δz is obtained when

$$4\sigma_s^2 = \frac{Re[\omega'']^2 t^2}{\sigma_s^2} \tag{5.29}$$

which inserted in Eq. 5.28 gives the minimised width

$$\Delta z = \frac{1}{\sqrt{2}} \frac{Re[\omega'']t}{\sigma_s} \tag{5.30}$$

The duration time, $T = \frac{\tilde{z}}{Re[\omega']_s}$, where the subscript s denotes storage and evaluation in $k = k_s$, is inserted in the minimised Δz to relate the width and centre, and thus obtain a condition on the stored light.

$$\begin{aligned} \tilde{z} &= \sqrt{2}\frac{\sigma_s Re[\omega']_s}{Re[\omega'']_s}\Delta z \\ &= \sqrt{2}\sigma_s k_s \Delta z \end{aligned} \tag{5.31}$$

The imposed condition $\tilde{z} \gg \Delta z$ is thus satisfied when

$$\sigma_s k_s \gg 1 \tag{5.32}$$

Using the real part of Eq. 5.3 to lowest order in δ imposes

$$\sigma_s \frac{d\delta}{\Omega^2} \gg 1 \tag{5.33}$$

which leads to the condition on d, that d is required to be large compared to δ: $d \gg \frac{1}{\delta}$.

This condition on d supports the analytical solution, since the comparison of the analytical and numerical solution in Fig. 4.1 yields that the two solutions converge for large d, ie. $d = 1000$. Thus the approximations made to solve the equations of motions analytically are good for a large optical depth, which is required for storage.

5.3 Numerical Solution of Travelling Wave

The condition on the optical depth to be large is examined by numerically deriving the time dependence of the propagating wave. The numerically solved propagating wave is compared with the stored wave to study the possibility of localisation of the two waves.

The numerical solutions of the propagating wave inside the atomic ensemble are made for $d = 100$ and $d = 1000$ with a numerical Fourier transform. This is done by using a Fast Fourier Transform of the analytical solution $\mathcal{S}_{out}(z)$. The transformed output field, $\mathcal{S}(k)$, is multiplied with the time dependence $e^{i\omega(k)T_t}$, where $\omega(k)$ is defined by the dispersion relation Eq. 5.19 and k is the wave vector. T_t is the time the wave propagates inside the ensemble. Since the transform is made numerically, and hence $\mathcal{S}_{out}(z)$ is discrete values, the zero frequency component is shifted.

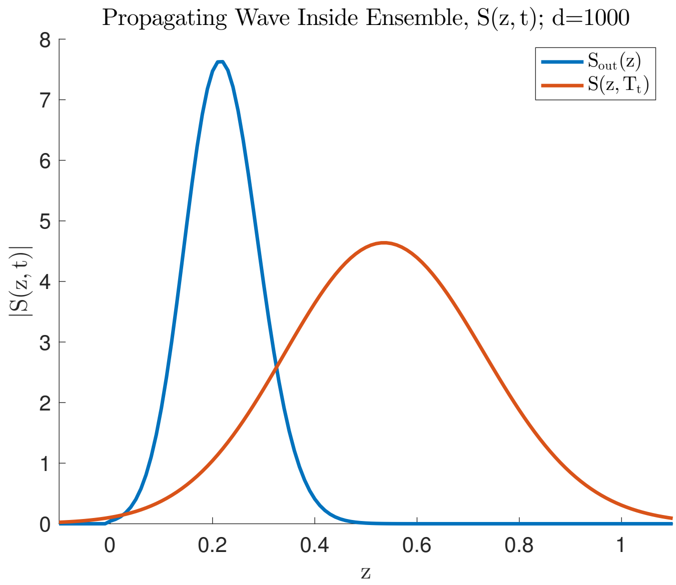

Figure 5.1: **Plot of the stored input field and numerically solved propagating wave inside the atomic ensemble for optical depth, d=1000.** The blue graph is the stored light, $\mathcal{S}_{out}(z)$ given by Eq. 5.13. The red graph is the propagating wave, which is obtained by numerically deriving the time evolution of the stored wave. The parameters of the input field are chosen to yield a stored wave with centre $z_s = 0.25$ and width $\sigma_s = 0.05$. The frequency, $\delta = 0.5$, and propagation time, $T_t = 0.0003$, are chosen to attain a wave, that has propagated a distance with minimum broadening.

The frequency, δ, is chosen small to decrease the attenuation of the stored wave and sufficiently large to move the wave. Time, T_t is chosen to make the wave propagate, however, the width increases with time. Thus δ and T_t are chosen to obtain a shape of the propagated wave that can be distinguished from the stored wave.

The numerical propagation inside the ensemble is made for two optical depths, d=100 (Fig. 5.2) and d=1000 (Fig. 5.1). In chapter 5.2 the condition of localisation of the

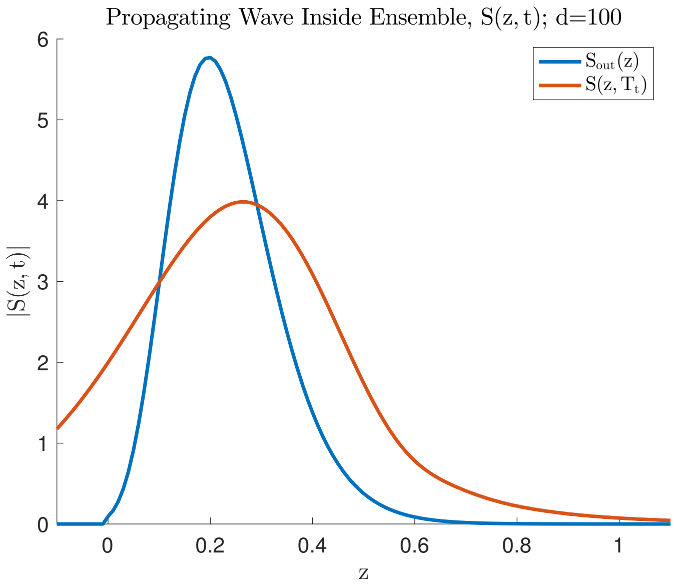

Figure 5.2: **Plot of the stored input field and the numerically solved propagating wave inside the atomic ensemble for optical depth, d=100.** The blue graph is the stored light, $\mathcal{S}_{out}(z)$ given by Eq. 5.13, with parameters chosen to yield a wave with centre $z_s = 0.25$ and width $\sigma_s = 0.05$. The red graph is the propagating wave, which is obtained by numerically deriving the time evolution of the stored wave. The parameters of the input field are chosen to yield a stored wave with centre $z_s = 0.25$ and width $\sigma_s = 0.05$. The frequency, $\delta = 0.3$, and propagation time, $T_t = 0.0004$, are chosen to attain a wave, that has propagated a distance with minimum broadening. Since $d = 100$ the parameters can not be chosen to obtain two distinguishable waves due to large broadening.

propagating wave required the optical depth to be large. Fig. 5.1 shows the propagating wave for large optical depth, ie. $d = 1000$, where δ and T_t are chosen to fit the stored and the propagating wave inside the ensemble. The two waves are distinguishable for $d = 1000$ as required, however for smaller optical depth, $d = 100$, the propagated wave can not be distinguished from the stored wave, since the propagating wave has broadened while the centres are only separated by a small distance compared to the separation for $d = 1000$. This separation for $d = 100$ is much smaller than the broadening, which is clear in Fig. 5.2.

A comparison of the figures of the stored and propagating wave supports the condition on the optical depth, d, imposed in chapter 5.2, since the optical depth is required to be large for storage of light in an atomic medium.

6 Conclusion

In this thesis the interface of light and an atomic medium is examined by an analytical derivation of the output field for storage and the propagating stored wave inside the atomic ensemble. The derivation is made by solving the equations of motion for the system and imposing the dispersion relation.

The analytical solution for storage yields the shape of the output field, with a width of the wave, a centre, a wave vector and a loss. The parameters for the input field are expressed as a function of chosen parameters of the stored light that ensures the possibility of the wave propagating inside the atomic ensemble. The attenuation of the stored wave is proportional to the frequency squared, δ^2, thus δ is required to be small. Furthermore the stored light must be localised during propagation, which yields the condition on the optical depth to be large. This condition supports the approximations made to solve the equations of motion, since the analytical and numerical solution of storage converge for large optical depth. Therefore, a large optical depth is assumed in the calculations. For further research the condition on the optical depth can be examined in detail to determine how the stored and and propagating wave depends on the optical depth explicitly, since this thesis is concerned with three different optical depths, $d = 10$, $d = 100$ and $d = 1000$.

The propagation of the wave is solved with the conditions of localisation of the stored and propagating wave and minimizing loss taken into account. These conditions are considered by assuming a large optical depth and a small frequency, δ. The propagation of the stored light is solved by deriving the time dependence of the stored input field. This yields a wave propagating with the velocity $Re[\frac{d\omega}{dk}\Big|_{k=k_s}]$, the real part of the derivative of the frequency, where the frequency, $\omega(k)$, is a complex function, and it is determined by the quadratic dispersion relation for a Λ-type atomic medium. The attenuation in time is found to be the imaginary part of $Im[\omega(k_s)]$. A comparison of the velocity of the propagating wave and the attenuation yields the condition of the detuning between the light frequency and the frequency of the atomic transition, $\Delta \gg \frac{1}{2}$.

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